Introduction to Quantum Mechanics II
Quiz 1

Name:

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Consider a two-level system, for example, a spin-1/2 system. Operators may be represented by 2×2 matrices in this system. Consider the following matrices:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Which two of these could represent a physical property or observable? Why? For the two which represent a physical property, are they compatible, in that their values can be simultaneously measured? Why? Are either of these compatible with

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}?$$

Solution:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

are physical properties because they are Hermitian, so their eigenvalues are real, $\sigma_x^\dagger = \sigma_x$, $\sigma_y^\dagger = \sigma_y$. The third matrix is not. They are incompatible because

$$[\sigma_x, \sigma_y] = 2i\sigma_z = 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \neq 0.$$

They are both compatible with the unit matrix

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

since all matrices commute with the unit matrix.