Introduction to Quantum Mechanics I
Quiz 9

Name:

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Consider spin-1/2, described by the three Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Start with the column vectors

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad a_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Show that these two vectors are interchanged by σ_x:

$$\sigma_x a_1 = a_2, \quad \sigma_x a_2 = a_1.$$

Show that the normalized eigenvectors of σ_x are

$$v_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad v_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

What are the corresponding eigenvalues? Show that these two vectors are interchanged by σ_z,

$$\sigma_z v_1 = v_2, \quad \sigma_z v_2 = v_1,$$

and compute the normalized eigenvectors of σ_z, u_1, u_2. What is the relation between these eigenvectors and the original vectors we started with, a_1 and a_2?

Solution:

By direct matrix multiplication

$$\sigma_x a_{1,2} = a_{2,1},$$

and

$$\sigma_x v_{1,2} = \pm v_{1,2}.$$

Then

$$\sigma_z v_{1,2} = v_{2,1},$$

and the eigenvectors of σ_z coincide with $a_{1,2}$.