Introduction to Quantum Mechanics I
Quiz 8

Name:

April 25, 2012

An atom with spin 1/2 is in a uniform magnetic field $\mathbf{B}$, where the magnetic field points along the z axis. The Hamiltonian for the system is

$$H = -\gamma B \frac{\hbar}{2} \sigma_z,$$

where γ is a constant, the gyromagnetic ratio. Using the Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t} \langle a', t| = \langle a', t| H,$$

solve for the probability amplitude

$$\langle +z, t| + z, t = 0\rangle,$$

and compute the probability that the atom, initially prepared in the state $|+z\rangle$, that is with $\sigma'_z = +1$, remains in that state after a time t in the magnetic field,

$$p(+z, t; +z, 0).$$

What is the probability that the spin will flip,

$$p(-z, t; +z, 0)?$$

Solution:

$$i\hbar \frac{\partial}{\partial t} \langle +z, t| + z, 0\rangle = \langle +z, t| \left(-\gamma B \frac{\hbar}{2} \sigma_z \right) | + z, 0\rangle = -\hbar\omega \langle +z, t| + z, 0\rangle,$$

where $\omega = \gamma B/2$. This is solved as

$$\langle +z, t| + z, 0\rangle = e^{i\omega t}\langle +z, 0| + z, 0\rangle = e^{i\omega t},$$

so

$$p(+z, t; +z, 0) = 1, \quad P(-z, t; +z, 0) = 0.$$