Introduction to Quantum Mechanics I
Quiz 7

Name:

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An atom with spin j is in a uniform magnetic field $\mathbf{B}$. The Hamiltonian for the system is

$$H = -\gamma \mathbf{J} \cdot \mathbf{B},$$

where γ is a constant, the gyromagnetic ratio, and $\mathbf{J}$ is the angular momentum operator for the atom. Assume that the magnetic field points in the z direction. Using the angular momentum commutation relations (you can use spin-1/2 if you like), and the Heisenberg equations of motion,

$$i\hbar \frac{d}{dt}\mathbf{J} = [\mathbf{J}, H],$$

show that

$$\frac{d}{dt}\mathbf{J} = \mathbf{J} \times \boldsymbol{\omega},$$

and express $\boldsymbol{\omega}$ in terms of γ and $\mathbf{B}$.

Hint: compute $\frac{d}{dt}J_x$, $\frac{d}{dt}J_y$, $\frac{d}{dt}J_z$.

Solution:

$$i\hbar \frac{d}{dt}\mathbf{J} = [\mathbf{J}, -\gamma J_z B],$$

so

$$\begin{aligned}
\frac{d}{dt}J_z &= 0, \\
i\hbar \frac{d}{dt}J_y &= -\gamma B[J_y, J_z] - \gamma B i\hbar J_x, \\
i\hbar \frac{d}{dt}J_x &= -\gamma B[J_x, J_z] = \gamma B i\hbar J_y.
\end{aligned}$$

so

$$\frac{d}{dt}\mathbf{J} = \gamma B(\hat{\mathbf{x}} J_y - \hat{\mathbf{y}} J_x) = \gamma B(\mathbf{J} \times \hat{\mathbf{z}}) = \mathbf{J} \times \boldsymbol{\omega},$$

$\boldsymbol{\omega} = \gamma \mathbf{B}$.