Introduction to Quantum Mechanics I
Quiz 5

Name:

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Using the 2×2 matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

factor

$$\frac{1+\sigma_x}{2} = \psi\psi^\dagger,$$

where ψ and $\psi^\dagger$ are two-component column and row vectors, respectively. Write ψ explicitly. What state of the spin-1/2 system does this eigenvector correspond to?

Solution:

$$\frac{1+\sigma_x}{2} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}(1,1),$$

thus

$$\psi_{+x} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

This corresponds to a state in which σ_x has the value $+1$.