First Examination Physics 3803 Introduction to Quantum Mechanics I

February 22, 2012

Instructions: Attempt all parts of this exam. If you get stuck on one part, assume an answer and proceed on. Do not hesitate to ask questions. Remember this is a closed book, closed notes, exam. *Good luck!*

1. (a) The transitions between states in the Bohr model of the hydrogen atom involve a change in the quantum number n of one unit,

 $n \to n-1, \quad n > 1.$

From this, what can you say about the angular momentum or spin of the light quantum (photon) emitted as a result?

- (b) Suppose the quantum relation between (de Broglie) wavelength and momentum p holds for photons as well as for atoms and electrons. Deduce the relation between energy E of the light quantum and the frequency of light, given that E = pc for either an electromagnetic wave or a massless particle, c being the speed of light.
- 2. Suppose A is a physical property assuming n possible values. Show that the most general function of A is a polynomial of degree n - 1 in A,

 $f(A) = \alpha_0 + \alpha_1 A + \alpha_2 A^2 + \ldots + \alpha_{n-1} A^{n-1},$

where the coefficients α_k are (complex) numbers.

3. In quantum statistical mechanics it is still true that for a system in thermal equilibrium at temperature T, the probability of finding the

system in a state of energy E is proportional to the Boltzmann factor $e^{-E/kT}$, where k is Boltzmann's constant and T is the absolute temperature. For a spin-1/2 atom in a magnetic field H,

$$E = -\boldsymbol{\mu} \cdot \mathbf{H} = -\gamma \mathbf{J} \cdot \mathbf{H} = -\gamma \frac{\hbar}{2} \sigma_z H$$

if **H** points along the z axis. Here γ is the gyromagnetic ratio, a characteristic property of the atom.

(a) Using the fact that σ_z takes on only two values, ± 1 , compute the thermal average

$$\langle \mu_z \rangle_T$$

in terms of the hyperbolic tangent function

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

(b) Why is

$$\langle \mu_x \rangle_T = \langle \mu_y \rangle_T = 0?$$

(c) What is the high-temperature limiting form? It should be in the form of Curie's law,

$$\langle \mu_z \rangle_T \propto \frac{1}{T}.$$

What is the constant of proportionality?