Introduction to Quantum Mechanics I
4th Homework Assignment
Due: Monday, February 20, 2012

February 13, 2012

1. Prove that the distributive property of multiplication holds for measurement symbols,

$$(|a'| + |a''|)|a'''| = |a'||a'''| + |a''||a'''|, \tag{1}$$

by showing that the two sides of this equation have the same meaning.

2. (a) Show that the sum of two measurement symbols

$$|a'| + |a''| \tag{2}$$

satisfies the equation

$$(A - a')(A - a'')\,(|a'| + |a''|) = 0. \tag{3}$$

Give a physical interpretation of this result.

(b) Thereby, from

$$1 = \sum_{a'} |a'|, \tag{4}$$

show that

$$\prod_{a'} |A - a'| = 0. \tag{5}$$

3. Verify that

$$\prod_{a'' \neq a'} \left(\frac{A - a''}{a' - a''} \right) = |a'| \tag{6}$$

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by verifying that all the required properties are satisfied:

$$(A - a')|a'| = 0, \tag{7a}$$
$$|a'||a'| = |a'|, \tag{7b}$$
$$|a'||a''| = 0, \quad a' \neq a'', \tag{7c}$$
$$\sum_{a'} |a'| = 1. \tag{7d}$$

4. A spin-1 system is characterized by a symbol with three possible values: A assumes the values $+1$, 0, -1.

   (a) Express these measurement symbols in terms of the measurement symbol for A; for example, show

   $$|0| = 1 - A^2. \tag{8}$$

   What are the corresponding expressions for $|+1|$ and $|-1|$?

   (b) Using the above, express $e^{i\lambda A}$ in terms of 1, A, and A^2, multiplied by trigonometric functions of the real number λ.