Introduction to Quantum Mechanics I
Second Homework Assignment
Due: Monday, February 6, 2012

January 30, 2012

1. Derive the virial theorem for a single particle, as follows. Start with Newton's law,
$$\mathbf{F} = m\frac{d\mathbf{v}}{dt}, \tag{1}$$
and by arguing that a total time derivative can be neglected for any bounded motion, prove that
$$-\overline{\mathbf{F}\cdot\mathbf{r}} = 2\bar{K}, \quad K = \frac{1}{2}mv^2, \tag{2}$$
where an overbar means a time average over a sufficiently long time interval. This is the general virial theorem. If $\mathbf{F}$ is derivable from a potential,
$$\mathbf{F} = -\boldsymbol{\nabla} V, \tag{3}$$
and $V = kr^\alpha$, establish the relation between $\bar{K}$ and $\bar{V}$. What, in particular, is the relation for the Coulomb potential,
$$V = -\frac{e^2}{r}? \tag{4}$$

2. Consider an atom entering a Stern-Gerlach apparatus, as shown in Fig. 1. Deflection upward begins as soon as the atom enters the inhomogeneous field. By the time the atom leaves the field, it has been deflected upward by a net amount Δz. Compute Δz for
$$\mu_z = 10^{-20}\,\mathrm{erg/G},\ \frac{\partial H_z}{\partial z} = 10^4\,\mathrm{G/cm},\ \ell = 10\,\mathrm{cm},\ T = \frac{mv_x^2}{k} = 10^3\,\mathrm{K}. \tag{5}$$

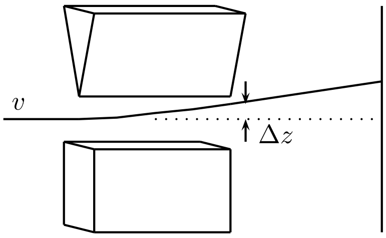

Figure 1: Deflection of an atom having a magnetic moment by an inhomogeneous magnetic field.

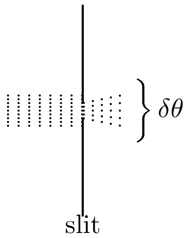

Figure 2: Diffraction of a beam of atoms by a small aperture in a screen.

3. A silver atom has mass (actually the stable isotopes are Ag^{107}, Ag^{109})
$$m = 108 \times 1.67 \times 10^{-24}\,\mathrm{g}, \tag{6}$$
and speed
$$v = 10^4\,\mathrm{cm/s}. \tag{7}$$
Compute the reduced de Broglie wavelength, $\lambda\!\!\!^-$, and the corresponding diffraction angle $\delta\theta$ when a beam of such atoms passes through a slit of width 10^{-2} cm. See Fig. 2. Compare this diffraction angle with the deflection angle produced in a Stern-Gerlach experiment.

4. What is the formula for the speed v of the electron in the nth Bohr orbit in hydrogen? Compute the numerical value of v/c for $n = 1$. Is the nonrelativistic approximation valid? Compute mc^2 for the electron in electron volts, eV, and compare with the energy of the ground state of hydrogen.

5. An object with mass 7.4×10^{22} kg moves in a circular orbit with radius 380,000 km, with a period of 29.5 days. (Such an object is the moon.)

What is n for this system? Why was the quantization of energy levels not first noticed in astronomical systems?

6. Improve on the Bohr atom calculation done in class by taking into account the small motion of the nucleus. You may proceed as follows:

   (a) Assume that the proton and electron revolve in circles about their common center of mass with total angular momentum $n\hbar$.

   (b) Compute the centripetal acceleration of the electron in terms of the electrical attraction produced by the proton.

   (c) Obtain an expression for the total energy.

   From this, find expressions for the energy of the nth orbit, E_n, and the radius, r_n of the nth orbit, in terms of n, e, $\hbar$, m, and M, the mass of the proton. How much deviation, numerically, is there from the formulas obtained by neglecting the proton motion?