Physics 3223
Lecture 17

October 13, 1999

1 The Nuclear Atom

J.J. Thomson, the discoverer of the electron (1897) proposed the "plum pudding" model of the atom, in which the negatively charged electrons were embedded in a uniformly charged positive background. This model had a certain attraction, but it failed to adequately account for the observed discrete spectral lines of light emitted by atoms.

This model was ruled out in 1910 by an experiment carried out by Geiger and Marsden, who observed that some alpha particles scattered off atoms underwent very large deflections. The Thomson model could only explain very small scattering angles: for a single atom, the average scattering angle was $\theta_1 \sim 0.01^\circ$, while if the alpha particle scattered off N atoms, it would be deflected by $\sqrt{N}\theta_1 \sim 1^\circ$. See the discussion in Krane, pp. 177–8. Ernest Rutherford, the director of the laboratory where Geiger and Marsden carried out their experiment, expressed his surprise eloquently:

> It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.

Rutherford concluded from this experiment that the positive charge of the atom was concentrated in a very small nucleus, having a insignificant volume compared to that of the atom, for then you could get very large scattering if the nucleus is hit very nearly head on.

Recall that the trajectory of a particle in a $1/r$ potential (such as that experienced by a planet orbiting the sun) is a conic section. For a scattering

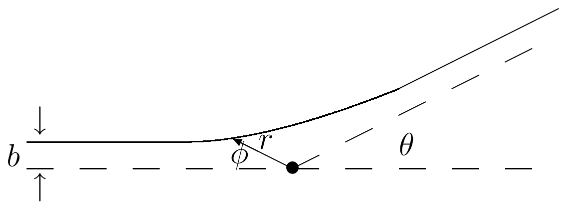

Figure 1: Hyperbolic trajectory of alpha particle scattering past a heavy nucleus.

event, that conic section is a hyperbola. For a projectile of charge ze scattering from a nucleus of charge Ze, the equation of that hyperbola is (see Fig. 1)

$$\frac{1}{r} = \frac{1}{b}\sin\phi + \frac{zZe^2}{8\pi\epsilon_0 b^2 K}(\cos\phi - 1). \tag{1}$$

Here b is the impact parameter—it measures the amount by which the collision fails to be head on. r and ϕ are the polar coordinates of the scattering particle, and K is the initial kinetic energy of that particle.

After the projectile has undergone scattering and is again far from the nucleus,

$$r \to \infty, \quad \phi \to \pi - \theta, \tag{2}$$

we have

$$\begin{aligned} 0 &= \frac{1}{b}\sin\theta + \frac{zZe^2}{8\pi\epsilon_0 b^2 K}(-\cos\theta - 1) \\ &= \frac{1}{b}2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \frac{zZe^2}{8\pi\epsilon_0 b^2 K}2\cos^2\frac{\theta}{2}, \end{aligned} \tag{3}$$

or

$$b = \frac{zZ}{2K}\alpha\hbar c\cot\frac{\theta}{2}. \tag{4}$$

1.1 Rutherford Scattering Cross Section

Note that for a given impact parameter b a definite scattering angle θ results. We now want a measure of the probability of scattering through an angle θ.

This is the differential cross section, the effective area the atom presents for scattering the alpha particles into the angular range θ to $\theta + d\theta$. We denote that area by $d\sigma$:

$$d\sigma = 2\pi b|db|, \tag{5}$$

which is just the area of an annular ring of radius b and width db. From the formula (4) for b, we have

$$|db| = \frac{zZ}{2K}\alpha\hbar c \csc^2\frac{\theta}{2}\frac{d\theta}{2}, \tag{6}$$

so the cross section becomes

$$d\sigma = \left(\frac{zZ}{2K}\right)^2 (\alpha\hbar c)^2 2\pi \frac{\cos\theta/2}{\sin^3\theta/2}\frac{d\theta}{2} \tag{7}$$

This is better defined in terms of the solid angle into which the scattered particles are thrown, which is the area of an annulus of radius $r\sin\theta$ and width $r\,d\theta$, divided by r^2:

$$d\Omega = 2\pi\sin\theta\,d\theta = 4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta. \tag{8}$$

In this way we get the Rutherford cross section,

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{zZ}{2K}\right)^2 (\alpha\hbar c)^2 \frac{1}{\sin^4\theta/2}. \tag{9}$$

To get the number of particles scattered into a unit solid angle, we simply multiply this by n, the number of atoms per unit volume,

$$n = \frac{N_A\rho}{M}, \tag{10}$$

where $N_A = 6.02 \times 10^{23}$ atoms/mole is Avogadro's number, ρ is the mass density of the material, and M is the molar mass , a mass in grams numerically equal to the molecular or atomic weight (thus, for ^{238}U, $M = 238$ g) and by t, the thickness of the sample:

$$\frac{dN(\theta)}{d\Omega} = \frac{nt}{4}\left(\frac{zZ}{2K}\right)^2 (\alpha\hbar c)^2 \frac{1}{\sin^4\theta/2} \tag{11}$$