Physics 506. Electrodynamics II.
Midterm Examination
Spring 2006

April 6, 2006

Instructions: This examination consists of two problems. If you get stuck on one part, assume a result and proceed onward. Do not hesitate to ask questions. GOOD LUCK!

1. (a) A particle with charge e moves along the z axis with constant speed v. Its coordinates are

$$x(t) = 0, \quad y(t) = 0, \quad z(t) = vt.$$

Construct the potentials in the Lorenz gauge by solving the differential equations satisfied by the potentials, noting that the only variables are x, y, and $z - vt$. Show that the result for the scalar potential is

$$\phi(x, y, z, t) = \frac{e/4\pi}{\sqrt{(z - vt)^2 + (1 - v^2/c^2)(x^2 + y^2)}}.$$

What is $\mathbf{A}$?

(b) Obtain the result of the preceding part by performing a Lorentz transformation of the Coulomb potential from a frame in which the charge e is at rest to one in which it is moving with velocity $\mathbf{v}$. [Hint: the components of the vector potential transform in the same way as the components of x^μ.]

(c) What is the power radiated by a particle moving with uniform velocity?

2. A nonrelativistic electron of charge e and mass m moves in a circular orbit under Coulomb forces produced by a proton. The average potential energy is related to the total energy by the virial theorem which here says

$$E = \frac{1}{2}\overline{V}.$$

Suppose, as it radiates, the electron continues to move in a circular orbit, of gradually decreasing radius, and calculate the power radiated, and thereby $-dE/dt$, as a function of E. Integrate this result, and find how long it takes for the energy to change from E_1 to E_2. In a finite time the electron reaches the center, so calculate how long it takes the electron to hit the proton if it starts from an initial energy of

$$E_{\text{Bohr}} = -13.6 \text{ eV}.$$

Use the following values:

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137},$$

$$\hbar = 6.58 \times 10^{-16} \text{eV s},$$

$$mc^2 = 0.511 \text{ MeV}.$$

An order of magnitude estimate is sufficient. This instability was one of the reasons for the discovery of quantum mechanics.