Homework Assignment # 3

Physics 5163: Grad. Stat. Mech.

Due: Wednesday, Feb. 9

Instructions:

Homework is due at the start of the Monday's class. You may turn it in at the lab, in my mailbox office, or after class on Monday.

Reading: I'll pass out an article on phase transitions in high energy physics. Don't worry about the stat mech parts of it. Just look for how it links to our study of the Landau theory of phase transitions.

Problems: Please solve the following:

1. Calculate:
   (a) $\left.\frac{\partial U}{\partial P}\right|_{G,N}$
   (b) $\left.\frac{\partial H}{\partial V}\right|_{T,N}$
2. Consider a system of a vapor coexisting with its liquid phase. Let q be the heat of vaporization per particle. Assume that the vapor obeys and ideal gas law, and that the volume of the liquid is very small. Show that

$$P(T) \approx e^{-q/kT}$$

   Hint: Start with the Clausius-Clapeyron equation.
3. Reichl, Problem 3.12. (*Hint:* Choose $\mathbf{n} = \hat{\mathbf{k}}$ and the sums are much, much easier.)
4. Reichl, Problem 3.13.

Computational Problems: You may find the computer a handy tool for the following problems.

1. The scaled Van der Waals equation of state is given by:

$$\tilde{P} + \frac{3}{\tilde{v}^2})(3\tilde{v} - 1) = 8\tilde{T}$$

   as in eq.3.44 in Reichl.
   (a) Determine the critical temperature, pressure and volume for the system in terms of these dimensionless variables.
   (b) Plot (on the same graph), $\tilde{P}(v)$ for values of $\tilde{T}$ greater than, equal to and less than, the critical temperature.
   (c) Determine an expression for the isothermal compressibility, κ. Plot $\kappa(\tilde{v})$ for temperatures above, just above, and below the critical temperature. Explain in physical terms what it means for κ to be
   - infinite,

- finite and positive,
- zero,
- negative.

citing explicit regions on your plots. (In particular, how do these values relate to the stability of a given phase and to the phase transition.)

(d) Calculate the chemical potential as we sketched in class, by directly integrating

$$\mu = \int_{v_0}^{v} v \; dp$$

determining the latter from the equation of state. Using the built-in command `ParametricPlot`, plot $\mu(v)$ vs $p(v)$ for $t = .2$ between the range $v = 1.05$ and $v = 15.0$. Interpret the shape of the graph, explaining what it means.

Questions: Answer the following questions.

1. A colleague of mine once claimed that Legendre and Fourier transforms were very similar since in former we replace x by dy/dx, and in the latter, we transform to a function of k, and the derivative operators transform as powers of k. Is this claim correct? Compare and contrast the Legendre and Fourier transforms. You may find it helpful to plot or calculate the two for different functions.