Homework Assignment # 2

Physics 5163: Grad. Stat. Mech.

Due: Monday, Jan. 31st

Instructions:

Homework is due at the start of the Monday's class. You may turn it in at the lab, in my mailbox office, or after class on Monday.

Reading: My presentation is taken from Callen and from Weinreich. Garrod's presentation is the same since it is based on Callen.

Please read Reichl, Chapter 3 on Phase transitions. We'll have a reading quiz on it sometime next week.

Problems: Please solve the following:

1. For the allowable fundamental equations in the last homework assignment, derive an expression for the free energy, F.
2. Consider a thermodynamic system where the differential of the internal energy is given by:
$$dU = T\,dS + f\,dx \tag{1}$$
where f is an intensive force variable and x is an extensive displacement variable.
   (a) Calculate all the Maxwell relations derivable from the second partials of U.
   (b) Calculate the following derivatives in terms of the natural variables of the system:
      i. $(df/dU)_T$
      ii. $(dU/dS)_f$
      iii. $(dx/df)_S$
   (c) Prove the following identities:
      i.
$$\left(\frac{dU}{dS}\right)_f + f\,\left(\frac{dT}{df}\right)_S = T$$
      ii.
$$\left(\frac{df}{dx}\right)_T = \left(\frac{df}{dx}\right)_S \left(\frac{df}{dT}\right)_x^2 \left(\frac{dT}{dS}\right)_x$$
3. In classical mechanics we extremize the action
$$A[q(t),\dot{q}(t)] = \int \mathcal{L}(q(t),\dot{q}(t))\,dt$$
where $\mathcal{L}$ is the Lagrangian:
$$\mathcal{L}(q,\dot{q}) = \frac{1}{2}m\,\dot{q}^2 - V(q)$$

which we extremize as a function of $q(t)$. (That is, the functions $q(t)$ and $\dot{q}(t)$ are an an infinite number of variables, and we extremize A with respect to the entire set.) In doing so we we obtain the standard Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

(a) Rather that chose $(q(t), \dot{q}(t))$ as our variables, let's assume we'd rather work with $(q(t), p(t))$, where

$$p(t) \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}}$$

Construct a new function which contains all of the same information as $\mathcal{L}$.

(b) Take the differential of this new function, and use it to determine the partial derivatives of the new function with respect to q and p.

(c) Repeat this process for the Lagranian:

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + \alpha\left(x\dot{y} - y\dot{x}\right)$$

where x and y are the standard Cartesian coordinates, and a "dot" denotes a time derivative i.e. $\dot{x} \equiv dx/dt$. The variable m is the mass, and α is a positive constant. Explicitly derive the Hamiltonian for this system.

4. In class we performed a Legendre transformation on the internal energy $U(S, V, N)$ to determine the free energy $F(T, V, N)$. However, we know that rather than minimize the energy, we can maximize the entropy, $S(U, V, N)$.

(a) Determine the *free entropy*, $S'(T, V, N)$, by performing a Legendre transform on $S(U, V, N)$. (Such Legendre transformed entropies are called *Massieu functions.*) Derive an expression for dS' in terms of its basic thermodynamic variables.

(b) Is S' minimized or maximized in equilibrium, for fixed T, V, and N? Prove your answer.