ASTRONOMY QUALIFYING EXAM

January 2009

Possibly Useful Constants

$L_\odot = 3.9 \text{ x } 10^{33}$ erg s^{-1}
$M_\odot = 2 \text{ x } 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \text{ x } 10^{10}$ cm
1 yr = $3.16 \text{ x } 10^{7}$ s
a = $7.56 \text{ x } 10^{-15}$ erg cm^{-3} K^{-4}
c= $3.0 \text{ x } 10^{10}$ cm s^{-1}
k = $1.38 \text{ x } 10^{-16}$ erg K^{-1}
e = $4.8 \text{ x } 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \text{ x } 10^{23}$ moles g^{-1}
G = $6.67 \text{ x } 10^{-8}$ g^{-1} cm^3 s^{-2}
m_e=$9.1 \text{ x } 10^{-28}$ g
h = $6.63 \text{ x } 10^{-27}$ erg s
1 amu = $1.66053886 \times 10^{-24}$ g

PROBLEM 1

The Hulse–Taylor binary pulsar consists of 2 neutron stars (NSs) orbiting each other with a period of about 8 hours. This system nicely exhibits a number of general relativistic effects. One of the neutron stars is a pulsar. We can time these pulses to mind–boggling accuracy to study the orbit of the system. The most spectacular effect is strong evidence for the emission of gravitational radiation from the system. Orbital timings show the system is losing orbital energy at a rate equal to the expected emission rate of gravitational radiation energy.

Assume the system consists of 2 identical NSs, each 1.4 times the mass of the Sun and each with a radius of 10 kilometers. Assume the current orbital period is exactly 8 hours.

a. (2 pts) What is the current separation of the centers of the NSs?

b. (3 pts) As the system loses energy due to emission of gravitational radiation, it shrinks and the NSs speed up in their orbit. At some time in the far future, the separation will decrease and the NSs will merge. This event will release an enormous flash of energy. Assuming the NSs remain spherical, calculate (ignoring general relativity) the orbital period when the NSs are just touching. (In reality the NSs will start to distort each other before this point is reached, but this calculation gives you an idea of the orbital period when the NSs are close to each other.)

c. (3 pts) Currently, the orbital period is observed to be shrinking at a rate of about $dP/dt = -2.4 \times 10^{-12}$ (i.e., P is shrinking by 2.4×10^{-12} seconds every second.) Calculate the change of the separation over one 8 hour period. (Hint: differentiate Kepler's third law with respect to time.)

d. (2 pts) Discuss the different forms of energy of the system and how and why they are changing. Explain, as precisely as you can, why the NSs speed up as the system loses energy.

PROBLEM 2

Consider a planet with the mass of Jupiter orbiting the nearby star α Centauri, a star very similar to the Sun.

a. (3 pts) α Centauri is at a distance of 1.3 pc (=4.238 ly) from the solar system. Suppose a planet orbits this star with an orbit of 5 AU in radius. As seen from the Earth, how large would the maximum angular separation between the star and planet be, in seconds of arc? Is this separation large enough that the two objects could be distinguished using ground-based telescopes?

b. (3 pts) Suppose that α Centauri has a radius of 1 $R_\odot$ and a surface temperature of 5800 K, and hence a luminosity of 1 $L_\odot$. If the hypothetical planet has the same radius as Jupiter (71,000 km), calculate the power striking its surface. Now suppose that the planet reflects all the incident starlight striking it from α Centauri, and that this radiation is reflected uniformly in all directions. How much visible light energy is reflected per second from the planet? What is the ratio of the planet's total reflected power to the power emitted by the star?

c. (3 pts) Now suppose that, like Jupiter, the hypothetical planet has an effective temperature for heat radiation of about 170 K. Find the wavelength for which the black-body emission of the planet is maximum, and find and evaluate an equation giving the ratio of the monochromatic power output of the planet and the star at this wavelength.

d. (1 pt) If you were going to try to detect the planet by direct imaging of the α Centauri system, would it be easier to do this using visible light or infrared? Explain.

PROBLEM 3

Consider Nova Scorpii 1994. Observations suggest a binary consisting of a subgiant star and a black hole.

a. (3 pts) Discuss how we know it is a binary and why the unseen companion is suspected to be a black hole. What are the assumptions?

b. (4 pts) An important discovery was the presence of alpha elements such as neon, oxygen, and silicon in the observed subgiant star. These elements are in proportions far above solar and can not have been produced in the subgiant star; they are only produced in explosive nucleosynthesis at temperatures of about 1 billion degrees. What does this suggest about the final evolution of the star that became a black hole?

c. (3 pts) Discuss a scenario for how this black hole formed. How might we test this scenario?

PROBLEM 4

Briefly discuss the relevance of the following terms to the study of the evolution of the Milky Way Galaxy.

a. (1 pt) stellar yield

b. (1 pt) initial mass function

c. (1 pt) metallicity

d. (1 pt) age-metallicity relation

e. (1 pt) abundance gradient

f. (1 pt) thin disk

g. (1 pt) G dwarf problem

h. (1 pt) infall

i. (1 pt) mass fraction

j. (1 pt) instantaneous recycling approximation

PROBLEM 5

In the context of supernova explosions, discuss the meaning and significance of the following terms.

a. (1 pt) homologous expansion

b. (1 pt) P Cygni profile

c. (1 pt) multiple scattering

d. (1 pt) pre–explosion images

e. (1 pt) ^{56}Ni

f. (1 pt) single degenerate

g. (1 pt) double degenerate

h. (1 pt) detonation

i. (1 pt) deflagration

j. (1 pt) deflagration to detonation transition

PROBLEM 6

The conversion of hydrogen to helium produces 6.4×10^{18} erg/g. For a star of mass M, hydrogen burning takes place in the core, which is a fraction f of the total mass. When it reaches the ZAMS the star has a homogenous composition (X, Y, Z), where X is the mass fraction of hydrogen, Y the mass fraction of helium, and Z the mass fraction of metals.

a. (1 pt) How much hydrogen will be converted to helium on the main sequence?

b. (1 pt) Express the energy released on the main sequence by the conversion of hydrogen to helium, E_{MS}, with M scaled to $\mathrm{M}_\odot$.

c. (2 pts) The main sequence lifetime t_{MS} depends on E_{MS} and L. Write t_{MS} in years where L and M are scaled to solar values. Assume $f = 0.15$.

d. (2 pts) An approximate mass-luminosity relation is:

$$\frac{M}{\mathrm{M}_\odot} = (\frac{L}{L_\odot})^{1/4}$$

Assume $X = 0.75$ and derive the relationship between t_{MS} and L in years. Derive the relationship between t_{MS} and M.

e. (4 pts) Estimate the main sequence lifetime of a star with $M_V = -2$, using the table provided. You will have to interpolate.

Allen's Astrophysical Quantities

$(b-y)_0 = -0.116 + 0.097c_1$ for an unreddened main-sequence B star,

$(b-y)_0 = 2.946 - 1.0\beta - 0.1\delta c_1\ (-0.25\delta m_1$ if $m_1 < 0)$ for A stars with $2.870 > \beta > 2.720$ and $\delta c_1 < 0.28$,

$(b-y)_0 = 0.222 + 1.11\Delta\beta + 2.7(\Delta\beta)^2 - 0.05\delta c_1 - (0.1 + 3.6\Delta\beta)\delta m_1$ for F stars with $2.630 < \beta < 2.720$ and $\delta c_1 < 0.28$, or $2.590 < \beta < 2.630$ and $\delta c_1 < 0.20$,

where $\Delta\beta = 2.720 - \beta$, $\delta c_1 = c_1 - c_{\mathrm{std}}$, $\delta m_1 = m_{\mathrm{std}} - m_1$; See Section 15.3.2 for c_{std} and m_{std}.

15.3.1 Calibration of MK Spectral Types [2, 21, 22]

Table 15.7 presents the absolute magnitude, color, effective surface temperature, and bolometric correction calibrations for the MK spectral classes. Table 15.8 gives the calibrated physical parameters for stars of the various spectral classes.

Table 15.7. *Calibration of MK spectral types.*

Sp	*M(V)*	*B − V*	*U − B*	*V − R*	*R − I*	T_{eff}	BC
MAIN SEQUENCE, V							
O5	−5.7	−0.33	−1.19	−0.15	−0.32	42 000	−4.40
O9	−4.5	−0.31	−1.12	−0.15	−0.32	34 000	−3.33
B0	−4.0	−0.30	−1.08	−0.13	−0.29	30 000	−3.16
B2	−2.45	−0.24	−0.84	−0.10	−0.22	20 900	−2.35
B5	−1.2	−0.17	−0.58	−0.06	−0.16	15 200	−1.46
B8	−0.25	−0.11	−0.34	−0.02	−0.10	11 400	−0.80
A0	+0.65	−0.02	−0.02	0.02	−0.02	9 790	−0.30
A2	+1.3	+0.05	+0.05	0.08	0.01	9 000	−0.20
A5	+1.95	+0.15	+0.10	0.16	0.06	8 180	−0.15
F0	+2.7	+0.30	+0.03	0.30	0.17	7 300	−0.09
F2	+3.6	+0.35	0.00	0.35	0.20	7 000	−0.11
F5	+3.5	+0.44	−0.02	0.40	0.24	6 650	−0.14
F8	+4.0	+0.52	+0.02	0.47	0.29	6 250	−0.16
G0	+4.4	+0.58	+0.06	0.50	0.31	5 940	−0.18
G2	+4.7	+0.63	+0.12	0.53	0.33	5 790	−0.20
G5	+5.1	+0.68	+0.20	0.54	0.35	5 560	−0.21
G8	+5.5	+0.74	+0.30	0.58	0.38	5 310	−0.40
K0	+5.9	+0.81	+0.45	0.64	0.42	5 150	−0.31
K2	+6.4	+0.91	+0.64	0.74	0.48	4 830	−0.42
K5	+7.35	+1.15	+1.08	0.99	0.63	4 410	−0.72
M0	+8.8	+1.40	+1.22	1.28	0.91	3 840	−1.38
M2	+9.9	+1.49	+1.18	1.50	1.19	3 520	−1.89
M5	+12.3	+1.64	+1.24	1.80	1.67	3 170	−2.73
GIANTS, III							
G5	+0.9	+0.86	+0.56	0.69	0.48	5 050	−0.34
G8	+0.8	+0.94	+0.70	0.70	0.48	4 800	−0.42
K0	+0.7	+1.00	+0.84	0.77	0.53	4 660	−0.50
K2	+0.5	+1.16	+1.16	0.84	0.58	4 390	−0.61
K5	−0.2	+1.50	+1.81	1.20	0.90	4 050	−1.02
M0	−0.4	+1.56	+1.87	1.23	0.94	3 690	−1.25
M2	−0.6	+1.60	+1.89	1.34	1.10	3 540	−1.62
M5	−0.3	+1.63	+1.58	2.18	1.96	3 380	−2.48