

ASTRONOMY QUALIFYING EXAM

August 2009

Possibly Useful Constants

$$L_{\odot} = 3.9 \times 10^{33} \text{ erg s}^{-1}$$

$$M_{\odot} = 2 \times 10^{33} \text{ g}$$

$$M_{bol\odot} = 4.74$$

$$R_{\odot} = 7 \times 10^{10} \text{ cm}$$

$$1 \text{ A.U.} = 1.50 \times 10^{13} \text{ cm}$$

$$1 \text{ yr} = 3.16 \times 10^7 \text{ s}$$

$$a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$c = 3.0 \times 10^{10} \text{ cm s}^{-1}$$

$$k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$e = 4.8 \times 10^{-10} \text{ esu}$$

$$1 \text{ fermi} = 10^{-13} \text{ cm}$$

$$N_A = 6.02 \times 10^{23} \text{ moles g}^{-1}$$

$$G = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}$$

$$m_e = 9.1 \times 10^{-28} \text{ g}$$

$$h = 6.63 \times 10^{-27} \text{ erg s}$$

$$1 \text{ amu} = 1.66053886 \times 10^{-24} \text{ g}$$

PROBLEM 1

You have a telescope with a mirror 2.5 meter in diameter. It is an $f/4$ system. Make sure you state units of your answers.

- a. (1 pt) As seen from Earth, the moon has an angular diameter of about 0.5 degree. What is the linear diameter of the image of the moon formed by this telescope?
- b. (1 pt) Explain what the term "diffraction limit" means in this context.
- c. (1 pt) At visible wavelengths (say 550 nm) what is the numerical value of the diffraction limit of this telescope?
- d. (3 pts) You equip the telescope with a CCD camera having 2048×2048 pixels, each pixel 20×20 microns in size. What is the solid angle (angular area) of the sky that would be imaged by each exposure of the CCD?
- e. (2 pts) You want to image the entire northern half of the sky with your telescope and CCD. How many images would be required to do this?
- f. (2 pts) Explain the term "quantum efficiency" (QE) as applied to light detectors. If you have a state of the art CCD system, what is its peak QE likely to be? In years BC (Before CCDs) astronomers used photographic emulsions to image the sky. What was the typical QE for the best of these emulsions?

PROBLEM 2

Suppose a comet having an orbital period of 100 years has a perihelion distance from the center of the Sun of $2 R_{\odot}$.

- a. (3 pts) Calculate the velocity of the comet as it passes perihelion.
- b. (3 pts) Assume that perihelion passage can be crudely described as a semi-circular trajectory past the sun with the comet's perihelion velocity. Estimate the time spent by the comet on this semi-circle. Consider this to be the duration of the perihelion passage.
- c. (3 pts) Assuming a constant rate of heating of the comet's surface equal to that occurring when the comet is at $2.0 R_{\odot}$ from the center of the sun, estimate the rate of loss of material from the heated side of the comet, in m s^{-1} of surface lost, if the comet has an albedo of $A = 0.1$, a density of $\rho = 1000 \text{ kg m}^{-3}$, and a heat of sublimation of $L = 2.8 \times 10^6 \text{ J kg}^{-1}$. Assume that all the heat supplied by the sun is used to sublime ices.
- d. (1 pt) Could a cometary nucleus of radius $r = 1.0 \text{ km}$ survive one perihelion passage under these circumstances?

PROBLEM 3

- a. (3 pts) Using the luminosity equation for radiative transport

$$\frac{L}{4\pi r^2} = -\frac{4}{3} \frac{acT^3}{\kappa\rho} \frac{dT}{dr},$$

use dimensional analysis to find an expression for the luminosity in terms of the mass, such that $L \propto M^\alpha$. Assume that the opacity, κ , for this star is entirely due to electron scattering and is independent of mass or density.

- b. (3 pts) Now assume that the actual luminosity of the star is

$$L = 4 \times 10^{33} (M/M_\odot)^\alpha \text{ ergs/sec}$$

Using the α you found in part (a) and assuming blackbody emission, calculate the effective temperature of a star that has $M = 3M_\odot$ and $R = 2R_\odot$. For this temperature, use the Saha equation

$$\log\left[\frac{N_{i+1}}{N_i}\right] = 2.5 \log T - \frac{5040}{T} \chi_i - 0.18$$

(where χ_i for H is 13.6 eV and $\log P_e = 0$ has been used) to determine the fraction of total H atoms that are ionized in the atmosphere of this star. Do you expect that such a star will have strong or weak H spectral lines, and why? What is the spectral type of this star?

- c. (4 pts) Calculate and compare the free-fall, Kelvin-Helmholtz and nuclear time scales (in years) for this star.

The free-fall acceleration is given by

$$\frac{|d^2 R|}{|dt^2|} = g.$$

Use dimensional analysis to get an expression for the free-fall time scale, τ_{f-f} , in terms of the average density $\bar{\rho}$. For our star of $M = 3 M_\odot$ and $R = 2 R_\odot$, calculate τ_{f-f} .

For the Kelvin-Helmholtz time scale, use the luminosity in part (b) and the total gravitational potential energy available to this $3M_\odot$ star.

For the nuclear time scale for the same star assume that only 10% of the star's mass contributes to energy generation and the luminosity is given in (b). (Assume .7 % mass loss in the nuclear conversion.)

In terms of stellar evolution do your numbers for these time scales make sense. If not, why not?

PROBLEM 4

- a. (3 pts) Contrast Pop. I and halo stars in the Milky Way Galaxy in terms of their general location, kinematics, and metallicity.
- b. (2 pts) Give a plausible model for the formation of the Milky Way which explains the differences discussed in part a).
- c. (2 pts) According to chemical evolution theory, why does the value of $[\text{Fe}/\text{O}]$ in any one location in the Galaxy tend to increase with time? If the initial mass function were flatter (higher fraction of massive stars), how would you expect that to affect the evolution of the local value of $[\text{Fe}/\text{O}]$? Explain.
- d. (3 pts) Using simple physics and assuming circular orbits for stars in the disk, derive a functional relation between surface density σ (mass/pc²), tangential (circular) velocity v , and galactocentric distance r . Show your work. What is implied about the behavior of surface density in regions of the Milky Way disk where the rotation curve is flat? Explain.

PROBLEM 5

There is strong observational evidence that our universe is dominated by dark energy today. For this problem assume a fixed matter density fraction Ω_m and a flat universe.

- a. (4 pts) Assume that dark energy has a constant equation of state, w_X . What is its density as a function of redshift?
- b. (1 pt) What is special about $w_X = -1$?
- c. (5 pts) In general, $w_X < 0$. How does the age of the universe depend on the value of w_X ?

PROBLEM 6

- (5 pts) Explain the role of degeneracy in the initiation of helium burning and carbon burning. Make sure to say which category of stars does what.
- (1 pt) Describe the nuclear burning processes when a star is on the giant branch.
- (2 pts) Describe the nuclear burning processes when a star is on the asymptotic giant branch (AGB).
- (2 pts) Explain what is happening in the core when the star is on the horizontal branch. Explain why the horizontal branch of 47 Tuc is more horizontal than that of the HR diagram of a generic cluster (see the figures on this page and the next page)



