Astronomy Qualifier
August 18, 2006

Problem 1:

a) [3 points] Imagine an H II region of pure hydrogen with density n atoms/cm^3 surrounding a single hot central star. Φ is the number of photons emitted by the star per second which are capable of photoionizing neutral hydrogen ($\lambda < 912$Å), while αn^2 is the number of recombinations per second per cm^3. If each photon results in a photoionization and the rate of photoionization equals the rate of recombination (photoionization equilibrium), find an expression for the radius of the ionized gas cloud in terms of n, Φ, and α.

b) [2 points] Find the diameter of an H II region in parsecs which surrounds an O star if Φ=10^{49}photons/s, $n = 10$ atoms/cm^3, and $\alpha = 2 \times 10^{-13}$.

c) [2 points] Repeat part (b) for the sun if $\Phi = 5 \times 10^{23}$ photons/s, while n and α remain the unchanged.

d) [3 points] Could an H II region around the sun in (c) be seen by an astronomer on α-Centauri (distance = 1.31 pc) using a telescope which can just barely resolve objects which are 1" in angular size?

Problem 2:

Consider a uniform slab of thickness T, in the z direction. The slab is sitting at the origin of the z-axis and extends to inifinity in the x and y directions. For this problem we will ignore emission from the slab itself. The slab is illuminated with a specific intensity I_0 at $z = 0$.

a) [2 points] Consider the case of only pure absorption (specified by an opacity κ). Write down the equation of radiative transfer along the z-axis and solve for the emergent intensity at $z = T$

b) [2 points] Now assume that we have both absorption and scattering (specified by a scattering opacity σ). Write down the equation for I along the z-axis in this case.

c) [3 points] Describe in words why the equation in part (b) is much harder to solve than the one in part (a)

d) [3 points] Describe the classical Lambda-iteration method for solving the scattering problem and the way in which it fails.

Problem 3:

This question is about the structure of, and physical conditions in, zero–age main–sequence (ZAMS) stars of 1 solar mass and 20 solar masses.

a) [1 point] Discuss the relative importance, for nuclear energy generation in the cores of these stars, of the CNO cycle and the proton–proton chain.

b) [2 points] At the microscopic level, which processes dominate the radiative opacity near the photospheres of these stars?

c) [2 points] Describe the relative importance of energy–transport mechanisms (radiation, convection, conduction) as a function of radius (from the center to the photosphere) of these stars.

d) [5 points] For each region in which a transport mechanism other than radiation dominates, explain why, in terms of physical quantities such as the nuclear energy generation rate, the temperature gradient, and the radiative opacity.

Problem 4:

The Robertson-Walker metric and the Einstein equations together give us the Friedmann equation:

$$(\frac{\dot{R}}{R})^2 + \frac{k}{R} = \frac{8\pi G\rho}{3}$$

as well as

$$(\frac{\ddot{R}}{R}) = -\frac{4\pi G}{3}(\rho + 3p)$$

a) [1 point] Define all the terms in the above equations and show that the Friedmann equation may be written as:

$$\frac{k}{H^2R^2} = \Omega - 1$$

where Ω is the ratio of the density to the critical density ρ_C.

b) [3 points] Derive the cosmic scale factor R as a function of time in a flat universe with only (constant) vacuum energy density, i.e. no matter or radiation.

c) [2 points] Write down the equations of state for radiation and for matter.

d) [4 points] Derive the matter density as a function of redshift in a flat universe.

Problem 5:

a) [4 points] Discuss in detail (and include an HR diagram) the evolution of a $5M_\odot$ star from its birth as a cloud of gas to its final fate. What is the final fate for stars with this mass and how do we know?

b) [3 points] How does the pre- and post-main sequence evolution differ for stars of $1M_\odot$ and $10M_\odot$? Assume that they have the same metallicity and explain (briefly) how each object ends up as a stellar remnant. Compare the evolution of a 1 $M_\odot$ star with solar metallicity to that of a 1 $M_\odot$ star with low metallicity (*i.e.* a Pop II star). Draw an HR diagram to illustrate the differences.

c) [3 points] Assuming that the luminosity of the $10M_\odot$ star is proportional to M^4, approximately how long does this star remain on the main sequence. What is the minimum time (approximately) for time on the Main Sequence, even for massive stars? When the $1M_\odot$ and $10M_\odot$ stars are on the main sequence, what process is producing energy in the core of each star? Where in these two stars do you expect to find regions of convection?

Problem 6:

A star has a surface temperature 0.55 times that of the Sun, and a diameter 0.35 times that of the Sun.

a) [2 points] What is the bolometric magnitude of the star?

b) [2 points] The star has a bolometric correction of -2.7. If the star is at a distance of 200 pc from us, what is its apparent V magnitude? (Think carefully about signs!)

c) [1 points] The star has a mass 0.2 times that of the Sun. Briefly describe its internal structure (which parts are radiative, which convective)?

d) [3 points] Estimate the lifetime of the star in years (to within a factor of 2). Work from the lifetime of the Sun (which you should know), the power of the star as compared to the Sun, and the fuel supply of the star as compared to the Sun. Make sure you take into account any differences in the structure of the star and the Sun.

e) [2 points] What is the approximate spectral type of this star? Are the hydrogen Balmer lines stronger or weaker in this star than in the Sun? Carefully explain what a Balmer line is and the physics behind your answer.