PHYS 3803: Quantum Mechanics I
Problem Set 7–Due March 24, 2021

Problem (1)

For a **free particle in one dimension**, the Schrödinger Equation is

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle \quad \text{with} \quad H = \frac{P^2}{2m}.$$

(a) Show that the wave function $\phi(p)$ in the momentum space or the $p-$basis is

$$\phi(p) = \langle p|\Psi\rangle = Ne^{-(i/\hbar)\left(\frac{p^2}{2m}\right)t}$$

where N is the normalization constant.

(b) Find the wave function $\Psi(x,t) = \langle x|\Psi\rangle$ in the coordinate space or the $x-$basis by applying the Fourier transform

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{(i/\hbar)px}\phi(p)\, dp\ .$$

Problem (2) [Griffiths 2.19]

Find the probability current for the free particle wave function

$$\psi(x,t) = Ae^{i\left(kx-\frac{\hbar k^2}{2m}t\right)}\ .$$

What direction does the probability current flow?

Problem (3) [Griffiths 2.4]

Let us consider the infinite square well with the potential energy

$$V(x) = \begin{cases} 0, & \text{for } 0 \le x \le a, \text{ and} \\ \infty, & \text{otherwise.} \end{cases}$$

Calculate $\langle X\rangle$, $\langle P\rangle$, $\langle X^2\rangle$, $\langle P^2\rangle$, ΔX, and ΔP for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit.

Problem (4)

For a particle moving in an **infinite square well** of width $2a$ with the potential energy

$$V(x) = \begin{cases} 0, & \text{for } -a \le x \le a \text{ with } a > 0, \text{ and} \\ \infty, & \text{otherwise,} \end{cases}$$

its normalized wave function inside the well at time $t = 0$ is

$$\Psi(x,0) = C[\sin\frac{\pi x}{a} + \frac{1}{4}\cos\frac{3\pi x}{2a}]$$

and $\Psi(x,0) = 0$ for $x^2 \ge a^2$.

(a). Calculate the coefficient C.

(b). What is the wave function $\Psi(x, t)$?

(c). If a measurement of total energy is made, what are the possible results of such a measurement, and what is the probability to measure each of them?

(d). What is the expectation value of the energy $\langle E \rangle$?