PHYSICS 6433
Problem Set 7 – Due April 14, 2017

Problems (1): Projection Operators

We can define operators P_L and P_R that project out the left and right handed components of a spinor. Use γ_5 to define properly normalized projection operators that satisfy

(a) $P_L^2 = P_L$ and $P_R^2 = P_R$,

(b) $P_L P_R = 0$, and

(c) $P_L + P_R = 1$.

Problems (2): Weyl Representation or Chiral Representation

(a) Show that unitary transformations of the Weyl representation of the gamma matrices preserve the properties

$$\begin{aligned} S^{0i\dagger} &= -S^{0i} \quad \text{and} \\ S^{ij\dagger} &= S^{ij}\,, \end{aligned}$$

of the boost and rotation generators of the Lorentz group. We can choose

$$S^{\mu\nu} = \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

(b) Show that the Weyl and Dirac representations are related by a unitary transformation.

Problems (3): Peskin and Schroeder, Problem 3.2

Problems (4): Dirac Field Bilinears

We can choose $\Lambda_{\frac{1}{2}} = S(\Lambda)$ and $P = \gamma^0$ to represent Lorentz transformation and the parity operation on spinors. By investigating transformations under parity and the proper Lorentz transformation, demonstrate that

(a) $\bar{\psi}\psi$ transforms as a scalar,

(b) $\bar{\psi}\gamma^5\psi$ transforms as a pseudoscalar,

(c) $\bar{\psi}\gamma^\mu\psi$ transforms as a vector,

(d) $\bar{\psi}\gamma^\mu\gamma^5\psi$ transforms as a pseudovector or an axial vector, and

(e) $\bar{\psi}\sigma^{\mu\nu}\psi$ transforms as a second rank tensor.