PHYS 3803: Quantum Mechanics I
Problem Set 6–Due March 12, 2021

Problem (1)

Find the normalization constant N for

$$\psi(x) = N\, exp[-\frac{1}{2c\hbar}(x - \langle X\rangle)^2 + \frac{i}{\hbar}\langle P\rangle x]$$

such that

$$\int dx|\psi(x)|^2 = 1$$

where c is a constant.

Problem (2) [Griffiths 1.17]

A particle is represented at time $t = 0$ by the following wave function

$$\psi(x) = \begin{cases} A(a^2 - x^2) & \text{for } -a \le x \le a,\ a\rangle 0\text{,and} \\ 0, & \text{for } x^2 > a^2, \end{cases}$$

(a) Determine the normalization constant A.

(b) What is the expectation value of X at time $t = 0$?

(c) What is the expectation value of P at time $t = 0$?

(d) Find the the expectation value of X^2.

(e) Find the the expectation value of P^2.

(f) Find the uncertainty (ΔX) in X.

Problem (3) [Griffiths 3.31, Virial theorem]

Apply the Ehrenfest's Theorem for a quantum operator Ω

$$i\hbar\frac{d}{dt}\langle\Omega\rangle = \langle\psi|[\Omega, H]|\psi\rangle + i\hbar\langle\frac{\partial}{\partial t}\Omega\rangle$$

to show that

$$\frac{d}{dt}\langle XP\rangle = 2\langle T\rangle - \langle X\frac{dV}{dx}\rangle\ ,$$

where T is the kinetic energy and the Hamiltonian $H = T + V$. In a stationary state, the left hand side is zero so

$$2\langle T\rangle = \langle X\frac{dV}{dx}\rangle\ .$$

(4) Let us consider a particle bound in a delta function potential in one dimension

$$V(x) = -\alpha\delta(x), \quad \alpha > 0,$$

where α is a constant.

(a) Find the wave function that satisfies the equation of motion

$$\begin{aligned} H\psi(x) &= E\psi(x) \\ H &= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \end{aligned}$$

in terms of $k = (-2mE/\hbar^2)^{1/2}$, where E is the eigenvalue of H and $\psi(x)$ is continuous at $x = 0$.

(b) Calculate the average total energy $\langle E\rangle$, the average potential energy $\langle V\rangle$ and then the average kinetic energy $\langle T\rangle$ of this particle with the relation $\langle T\rangle = \langle E\rangle - \langle V\rangle$.

(c) Calculate $\langle T\rangle$ for the previous problem directly as

$$\int dx\psi^*(x)(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2})\psi(x)$$

and compare with the previous result.