

Physics 5403

Problem Set 6 – Due October 23, 2009

Problem 1

A one dimensional oscillator in its ground state at $t = -\infty$ is subject to a perturbation

$$H_1(t) = V(t) = (ce^{-\alpha|t|} \cos \omega t) P$$

where c, α, ω are constants. What is the probability that the system would be found in the first excited state at $t = \infty$.

N.B. The oscillation frequency ω_c is different from ω .

Problem 2

Let us assume that a system is subjected to a constant perturbation between the interval $0 \leq t \leq t_0$ with the initial state $|\alpha\rangle$ and the final state $|\beta\rangle$. Thus

$$H_1(t) = V_0 = \text{constant in time} .$$

- (a) Find the probability ($P_{\alpha\beta}$) for the system.
- (b) Sketch the probability ($P_{\alpha\beta}$) as a function of $\omega_{\beta\alpha} = (E_\beta - E_\alpha)/\hbar$.

Problems 3: Ground State of The Helium Atom

The Hamiltonian of the Helium atom is

$$\begin{aligned} H &= H_0 + H_1 \\ H_0 &= -\frac{\hbar^2}{2m} \vec{\nabla}_1^2 - \frac{\hbar^2}{2m} \vec{\nabla}_2^2 - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} \\ H_1 &= \frac{e^2}{r_{12}} \end{aligned}$$

where $Z = 2$, m is the mass of the electron; \vec{r}_1 and \vec{r}_2 are the coordinates of the two electrons, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, and $r_i = |\vec{r}_i|$, $i = 1, 2$.

The ground state of H_0 with $Z = 2$ is given by

$$\psi_0(r_1, r_2) = \frac{8}{\pi a_0^3} e^{-2(r_1+r_2)/a_0}$$

with

$$E_0^{(0)} = -\frac{8e^2}{2a_0} = -8 \text{ Ry.}$$

- (a). Find the first order change of the ground state energy $E_0^{(1)}$ for the Helium atom by applying stationary perturbation theory with

$$\lambda H_1 = \frac{e^2}{R_{12}} .$$

- (b). Find the ground state energy E_0 to the first order for the Helium atom.

N.B. The experimentally measured value is $E_0 = -78.6$ eV for the Helium atom.