PHYS 5213/4213: Nuclear and Particle Physics, Autumn 2021

Problem Set 6 – due October 27

The simplest non-cyclic group is usually called the four-group or the dihedral group D_2. This group has four elements $\{e, a, b, c\}$ that follow the multiplication rules in Table 1.

Table 1: Multiplication table of the D_2 group.

e	a	b	c
a	e	c	b
b	c	e	a
c	b	a	e

1). Is D_2 an Abelian group? Justify your answer.

2). What are the subgroups of D_2?

3). In the two-dimensional Euclidean space, consider a group of four elements:
e = the identity, a = the reflection with respect to (w.r.t.) the horizontal direction,
b = the reflection w.r.t. the vertical direction, c = the rotation by π around the origin.
Find 2×2 matrices representing these operations with the basis vectors $u_1 = \hat{x}$ and $u_2 = \hat{y}$, then show that these matrices form a representation of D_2.

The Quark Model of Hadrons

4). There are six quarks and six leptons in the Standard Model. The quarks and the leptons are point-like fermions with spin quantum number of $\frac{1}{2}$.

Table 2: Properties of quarks, Q = electric charge, I = isosipn, B = baryon number, and the flavor quantum numbers: s = strangeness, c = charm, b = bottom, and t = top.

Quark	Mass (GeV)	Q (e)	B	I	s	c	b	t
u	5×10^{-3}	$+2/3$	$1/3$	$1/2$	0	0	0	0
d	9×10^{-3}	$-1/3$	$1/3$	$1/2$	0	0	0	0
s	0.2	$-1/3$	$1/3$	0	-1	0	0	0
c	1.4	$+2/3$	$1/3$	0	0	$+1$	0	0
b	4.8	$-1/3$	$1/3$	0	0	0	-1	0
t	175	$+2/3$	$1/3$	0	0	0	0	$+1$

There are two classes of hadrons: mesons and baryons,

- every meson can be described as a bound state of a quark and an antiquark,
- every baryon can be considered as constructed out of three quarks.

The quark contents are presented for a representative set of relatively long-lived hadrons in Table 3. Find the electric charge (Q), the baryon number (B), the strangeness (S), and the strong hypercharge $Y = B + S$ for every hadron in this table, then apply the Gell-Mann-Nishjima relation

$$Q = I_3 + \frac{Y}{2}$$

to find the third component of isospin I_3. All these quantum numbers are additive.

Table 3: A representative set of relatively long-lived hadrons.

Hadron	Quarks	Q (e)	B	S	Y	I_3
π^+	$u\bar{d}$					
π^0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$					
π^-	$d\bar{u}$					
K^+	$u\bar{s}$					
K^0	$d\bar{s}$					
$\bar{K}^0$	$s\bar{d}$					
K^-	$s\bar{u}$					
η	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$					
η'	$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$					
p	uud					
n	udd					
Σ^+	uus					
Σ^0	$\frac{1}{\sqrt{2}}(ud + du)s$					
Σ^-	dds					
Λ^0	$\frac{1}{\sqrt{2}}(ud - du)s$					
Ξ^0	uss					
Ξ^-	dss					
Ω^-	sss					