PHYS 3803: Quantum Mechanics I

Problem Set 5 – Due March 5, 2021

Problem (1)

Show that

$$\frac{d}{dx}\theta(x) = \delta(x),$$

where $\theta(x)$ is the step function such that

$$\theta(x) = \begin{cases} 1, & \text{for } x > 0, \text{ and} \\ 0 & \text{for } x < 0. \end{cases}$$

Hint: Consider a regular function that vanishes at infinity, $f(x) \to 0$ as $x \to \infty$.

Problem (2)

A generalized function $\sigma(x)$ has the property that for any test function $f(x)$

$$\int_{-\infty}^{+\infty} \sigma(x) f(x) dx = a[f'(a) - f'(-a)]$$

where $f'(x) = df/dx$. Determine $\sigma(x)$.

Problem (3)

Calculate the Fourier transform $g(\vec{k})$ of the function

$$f(\vec{r}) = N e^{-r^2/a^2}, \quad r = |\vec{r}|$$

in the 3-dimensional space. Find the normalization constant N for $f(\vec{r})$ and an exact expression of $g(\vec{k})$.

Problem (4)

Let us consider the following operators on a Hilbert space V^3:

$$L_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad L_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad L_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Find the eigenvalues ℓ_z and normalized eigenvectors $|\ell_z\rangle$ of L_z.

(b) Take the state in which $\ell_z = 1$. In this state, what are $\langle L_x \rangle, \langle L_x^2 \rangle$, and ΔL_x.

(c) Find the eigenvalues and the normalized eigenvectors of L_x in ℓ_z basis.

(d) If the particle is in the state with $\ell_z = -1$, and L_x is measured, what are the possible outcomes and their probabilities?