Physics 5403

Problem Set 4 – Due September 25, 2009

(1). Brillouin-Wigner Perturbation Theory.

Let us assume a completely nondegenerate system with

$$\begin{aligned} H_0|\psi_n^{(0)}\rangle &= E_n|\psi_n^{(0)}\rangle \\ H &= H_0 + \lambda H_1 \\ H|\psi_n\rangle &= E|\psi_n\rangle \text{ and} \\ |\psi_n\rangle &= |\psi_n^{(0)}\rangle + \lambda|\psi_n^{(1)}\rangle + \lambda^2|\psi_n^{(2)}\rangle + O(\lambda^3)\ . \end{aligned}$$

Derive perturbation equations by expanding only $|\psi_n\rangle$ in powers of λ but not E. Solve for the state vector up to first order in λ and the energy eigenvalues (E) up to second order in λ.

N.B. *This is not exactly our second order perturbation result. If we iterate E, we would get back the familiar result.*

(2). In the non-relativistic approximation, evaluate the perturbation of the first two energy levels of a hydrogen atom placed in an electric field $\vec{E}$ (the Stark effect) with the perturbation operator

$$\lambda H_1 = -\vec{d}\cdot\vec{E} = -eEZ\ .$$

where we have chosen the $z-$axis along the direction of $\vec{E}$.

(3). A two dimensional isotropic oscillator is subjected to a time independent perturbation H_1 whose matrix elements vanish between two states that have the same parity in either X or Y (e.g. $H_1 = \alpha XY$, $\alpha =$ constant).

(a) What is the degeneracy of the unperturbed state with eigenvalue $E^{(0)} = 3\hbar\omega$.

(b) In the Dirac notation, list the matrix elements of H_1 between eigenvectors belonging to this value, which do not vanish from symmetry.

(c) What is the first order change in the energy level in terms of these matrix elements?