# PHYS 3803: Quantum Mechanics I

## Problem Set 4 – Due February 26, 2021

1). The Fourier transform of $f(x)$ is defined as

$$g(k) \equiv \frac{1}{\sqrt{2\pi}} \int e^{-ikx} f(x)\, dx\ .$$

Express the Fourier transforms $G(k)$ of the following functions $F(x)$ in terms of $g(k)$.

(a) $f(x+a)$

(b) $f^*(x)$

(c) $f(-x)$

(d) $e^{i\mu x} f(x)$ where $\mu$ is real.

(e) $df(x)/dx$

2). Apply two dimensional polar coordinates and Show that

$$I(\alpha) \;=\; \int_{-\infty}^{\infty} e^{-\alpha x^2}\, dx \;=\; \sqrt{\frac{\pi}{\alpha}}\ ,$$

which is the Gaussian integral. Then find the Fourier transform of $f(x) = e^{-\alpha^2 x^2/2}$.

3). (Griffiths 3.23) The Hamiltonian for a certain two-level system is

$$H = \epsilon(|e_1\rangle\langle e_1| - |e_2\rangle\langle e_2| + |e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|)$$

where $|e_1\rangle$ and $|e_2\rangle$ are the orthonormal basis vectors and $\epsilon$ is a number with the dimension of energy. Find the matrix $H$ as well as its eigenvalues and eigenvectors as a linear combination of $|e_1\rangle$ and $|e_2\rangle$.

4). (Griffiths 3.30) Let us consider a wave function

$$\psi(x) = \frac{A}{x^2+\alpha^2}\ , (-\infty < x < +\infty)$$

at $t = 0$, for constants $A$ and $\alpha$.

(a) Determine $A$, by normalizing $\psi(x)$.

(b) Find $\langle X\rangle$, $\langle X^2\rangle$, and $\Delta X$.

(c) Find the momentum space wave function $\Phi(p)$.

(d) Use $\Phi(p)$ to calculate $\langle P\rangle$, $\langle P^2\rangle$, and $\Delta P$.

(e) Check the Heisenberg uncertainty principle for this state.