PHYS 3803: Quantum Mechanics I

Problem Set 3 – Due February 17, 2021

1). (Griffiths 3.22) Consider a three-dimensional vector space spanned by an orthonormal basis $|e_1\rangle$, $|e_2\rangle$, $|e_3\rangle$. Two ket vectors are given by

$$\begin{aligned} |\alpha\rangle &= i|e_1\rangle - 2|e_2\rangle - i|e_3\rangle\ , \\ |\beta\rangle &= i|e_1\rangle + 2|e_3\rangle\ . \end{aligned}$$

(a) Construct $\langle\alpha|$ and $\langle\beta|$ in terms of the dual basis $\langle e_1|$, $\langle e_2|$, $\langle e_3|$.

(b) Find $\langle\alpha|\beta\rangle$ and $\langle\beta|\alpha\rangle$, and confirm that $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$.

(c) Find all nine matrix elements $\Omega_{mn}, m, n = 1, 2, 3$ of the operator $\Omega = |\alpha\rangle\langle\beta|$ in this basis and construct that matrix Ω. Is it Hermitian?

2). Find the eigenvalues and the normalized eigenvectors for a matrix operator Ω,

$$\Omega = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix} .$$

3). Apply the normalized eigenvectors in Problem 2; construct a unitary matrix U. Then diagonalize Ω with the unitary matrix U.

4). Determine the value of the constant B for

$$t_b(x) = \begin{cases} 0, & \text{for } x^2 > b^2, \text{ and} \\ B(b - |x|) & \text{for } x^2 \le b^2,\ b > 0, \end{cases}$$

such that

$$\lim_{b\to 0} t_b(x) = \delta(x).$$