Physics 5403

Problem Set 3 – Due September 18, 2009

(1). A one-dimensional simple harmonic oscillator is subjected to a perturbation

$$\lambda H_1 = bX$$

where b is a real constant.

(a) Calculate the energy shift of the ground state to lowest nonvanishing order.

(b) Find the exact solution and compare it with your result obtained in (a).

Hint: Apply $X = \sqrt{\hbar/(2m\omega)}(a + a^\dagger)$.

(2). For $H = H_0 + \lambda V$, the expansion for a perturbed ket vector goes as follows

$$\begin{aligned} |n\rangle &= |n^{(0)}\rangle + \lambda \sum_{k\neq n} \frac{|k^{(0)}\rangle V_{kn}}{(E_n^{(0)} - E_k^{(0)})} \\ &\quad + \lambda^2 \left(\sum_{k\neq n} \sum_{\ell\neq n} \frac{|k^{(0)}\rangle V_{kn} V_{\ell n}}{(E_n^{(0)} - E_k^{(0)})(E_n^{(0)} - E_\ell^{(0)})} - \sum_{k\neq n} \frac{|k^{(0)}\rangle V_{kn} V_{nn}}{(E_n^{(0)} - E_k^{(0)})^2} \right) + \cdots \, , \\ V_{kn} &\equiv \langle k^{(0)}|V|n^{(0)}\rangle \, . \end{aligned}$$

Apply the above formula for a nondegenerate time-independent perturbation theory, and evaluate the probability of finding the unperturbed eigenstate ($|n^{(0)}\rangle$) in a perturbed energy eigenstate ($|n\rangle$). Solve this problem up to the second order in λ.

(3). Let us consider a one-dimensional simple harmonic oscillator with an additional perturbation

$$H_1 = \frac{1}{2}\epsilon m\omega^2 X^2$$

where ϵ is an infinitesimal parameter.

Show that

$$\begin{aligned} V_{00} &= \frac{\epsilon\hbar\omega}{4} \\ V_{20} &= \frac{\epsilon\hbar\omega}{2\sqrt{2}} \end{aligned}$$

and that all other matrix elements V_{k0} vanish with

$$V_{k0} = \langle k^{(0)}|H_1|0^{(0)}\rangle \, .$$