PHYS 6213: Advanced Particle Physics

Problem Set 3 – due March 11

Problem 1: The QCD Lagrangian

In quantum chromodynamics (QCD), the Lagrangian density is often expressed as

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2\xi} \left(\partial_\mu G^{a\mu} \right)^2 + \bar{\psi} i \gamma^\mu D_\mu \psi$$

whereG^a_μ is a gluon field, ψ is a quark field, the field strength tensor is

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu \,,$$

T^a are $SU(3)$ generators, and the covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + i g_s G^a_\mu T^a \,, \quad a = 1, \cdots, 8 \,,$$

with g_s being the strong coupling.

(a) Draw the Feynman diagram and describe the Feynman rule for the gluon propagator in the R_ξ gauge with results from Problem 1 of Problem Set 2.

(b) Draw the Feynman diagram and derive the Feynman rule (Λ^a_μ) for $Gq\bar{q}$ interactions with

$$\begin{aligned} iT_{\beta\alpha} &= \langle g(p_3)| - i \int \mathcal{H}_I(x)\, d^4x \,|u(p_1)\bar{u}(p_2)\rangle \\ &= (2\pi)^4 \delta^4(p_3 - p_\alpha) \bar{v}(p_2) \left(\Lambda_\mu\right) u(p_1) \left(\epsilon^\mu\right)^* \,. \end{aligned}$$

PROBLEM 2: Matrix Element of $u\bar{u} \to t\bar{t}$

Draw Feynman diagrams and evaluate the matrix element squared $\langle |M|^2 \rangle$ for $u(p_1)\bar{u}(p_2) \to t(p_3)\bar{t}(p_4)$, summed over all spins and colors as well as averaged over initial spins and colors in the Feynman gauge for gluons with $\xi = 1$.

PROBLEM 3: Cross Section of $q\bar{q} \to t\bar{t}$ at LHC

Evaluate the cross section $pp \to t\bar{t} + X$ as well as from quark antiquark annihilation alone $q\bar{q} \to t\bar{t}$, and gluon fusion alone $gg \to t\bar{t}$, at the Large Hadron Collider for collider energy $\sqrt{S} = 13, 14$, and 100 TeV, by using MadGraph with $\mu_F = \mu_R = m_t = 173.2$ GeV, where $\mu_F =$ the factorization scale and $\mu_R =$ the renormalization scale.