Problem 1: The QCD Lagrangian

In quantum chromodynamics (QCD), the Lagrangian density is often expressed as

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2\xi} (\partial_\mu G^{a\mu})^2 + \bar{\psi} i \gamma^\mu D_\mu \psi \]

where \( G^{a\mu} \) is a gluon field, \( \psi \) is a quark field, the field strength tensor is

\[ F_{\mu\nu}^a = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu, \]

\( T^a \) are \( SU(3) \) generators, and the covariant derivative is

\[ D_\mu = \partial_\mu + ig_s G^a_\mu T^a, \quad a = 1, \ldots, 8, \]

with \( g_s \) being the strong coupling.

(a) Draw the Feynman diagram and describe the Feynman rule for the gluon propagator in the \( R_\xi \) gauge with results from Problem 1 of Problem Set 2.

(b) Draw the Feynman diagram and derive the Feynman rule \( (\Lambda^a_\mu) \) for \( Gq\bar{q} \) interactions with

\[ iT_{\beta\alpha} = \langle g(p_3) | - i \int \mathcal{H}_I(x) d^4x | u(p_1) \bar{u}(p_2) \rangle = (2\pi)^4 \delta^4(p_3 - p_\alpha) \bar{v}(p_2) (\Lambda_\mu) u(p_1) (\epsilon^\mu)^*. \]

PROBLEM 2: Matrix Element of \( u\bar{u} \to t\bar{t} \)

Draw Feynman diagrams and evaluate the matrix element squared \( \langle |M|^2 \rangle \) for \( u(p_1)\bar{u}(p_2) \to t(p_3)\bar{t}(p_4) \), summed over all spins and colors as well as averaged over initial spins and colors in the Feynman gauge for gluons with \( \xi = 1 \).

PROBLEM 3: Cross Section of \( q\bar{q} \to t\bar{t} \) at LHC

Evaluate the cross section \( pp \to t\bar{t} + X \) as well as from quark antiquark annihilation alone \( q\bar{q} \to tt \), and gluon fusion alone \( gg \to tt \), at the Large Hadron Collider for collider energy \( \sqrt{S} = 13, 14, \) and 100 TeV, by using MadGraph with \( \mu_F = \mu_R = m_t = 173.2 \) GeV, where \( \mu_F \) = the factorization scale and \( \mu_R \) = the renormalization scale.