

Physics 5403

Problem Set 2 – Due September 11, 2009

(1). A quantum mechanical state $|\phi\rangle$ is known to be a simultaneous eigenstate of two Hermitian operators Ω_1 and Ω_2 that anti-commute,

$$\{\Omega_1, \Omega_2\} = \Omega_1\Omega_2 + \Omega_2\Omega_1 = 0 .$$

What can you say about the eigenvalues of Ω_1 and Ω_2 with a common eigenvector $|\phi\rangle = |\omega_1, \omega_2\rangle$? Illustrate our point with the parity operator (Π) and the momentum operator (P).

(2). Suppose a spinless particle is bound to a fixed center by a potential $V(\vec{r})$ so asymmetrical that no energy level is degenerate.

(a) Apply time reversal invariance with the time reversal operator T and prove that the expectation value of the orbital angular momentum operator is

$$\langle L \rangle = 0$$

for any energy eigenstate $|\psi\rangle$.

(b) If the wave function of such an eigenstate is expanded as

$$\psi(r, \theta, \phi) = \langle r, \theta, \phi | \psi \rangle = \sum_{\ell} \sum_m F_{\ell, m}(r) Y_{\ell}^m(\theta, \phi),$$

what kind of phase restrictions do we obtain on $F_{\ell, m}$?

Hint: Let's take $|\ell, m\rangle$ to be the eigenvectors of orbital angular momentum.

(3). The Hamiltonian for a spin 1 system is given by

$$H = aS_z^2 + b(S_x^2 - S_y^2) .$$

(a) Find the matrix operators for S_x , S_y , and S_z , in the s_z basis.

(b) Find the Hamiltonian as a 3×3 matrix operator.

(c) Find the eigenvalues and normalized eigenvectors for the Hamiltonian.

(d) Is this Hamiltonian invariant under time reversal?

(e) How do the normalized eigenstates you obtained transform under time reversal?