PHYS 3803: Quantum Mechanics I

Problem Set 2 – Due February 10, 2021

1). An operator Ω is linear if

$$\Omega(a\phi(x) + b\psi(x)) = a(\Omega\phi(x)) + b(\Omega\psi(x)),$$

where $\phi(x)$ and $\psi(x)$ are functions of x while a and b are complex numbers. Determine which of the following operators are linear.

(a) $\Omega\phi(x) = \phi(-x)$,

(b) $\Omega\phi(x) = \phi(x + c)$,

(c) $\Omega\phi(x) = \phi(x) + c$,

(d) $\Omega\phi(x) = \int_{-\infty}^{\infty} K(x, y)\phi(y)dy$,

where $c =$ constant and $K(x, y) = K^*(y, x)$.

2). Which of the following operators are Hermitian?

(a) $\Omega\phi(x) = \phi(x + a)$,

(b) $\Omega\phi(x) = \phi^*(x)$,

(c) $\Omega\phi(x) = \phi(-x)$,

(d) $\Omega\phi(x) = \int_{-\infty}^{\infty} K(x, y)\phi(y)dy$,

where $a =$ constant, $K(x, y)$ is real, and $K(x, y) = -K(y, x)$.

3). If A, B and C are Hermitian Operators, determine if the following combinations are Hermitian:

(a) $A + B$,

(b) $\frac{1}{2i}[A, B]$,

(c) $ABC - CBA$,

(d) $A^2 + B^2 + C^2$.

4). Find B_α such that

$$\lim_{\alpha\to\infty} B_\alpha e^{-\alpha r} = \delta^3(\vec{r}).$$