PHYS 6213: Advanced Particle Physics

Problem Set 2 – due February 24

Problem 1: Photon propagator in the R_ξ gauge

In QED, the free field generating functional in the covariant gauge is given as

$$Z_0[J] = \int \mathcal{D}A_\mu \, e^{iS[A_\mu]} = \int \mathcal{D}A_\mu \, e^{i\int d^4x \, [\mathcal{L}_0 + J^\mu A_\mu]}$$

where J is an external source and

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 \, .$$

(a) Show that the action can be rewritten as

$$S[A^\mu] = \int d^4x \, \left[\frac{1}{2}A_\mu K^{\mu\nu} A_\nu + J^\mu A_\mu\right]$$

with

$$K^{\mu\nu} = g^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu$$

where ξ is an arbitrary gauge fixing parameter.

(b) If we define the Green's function $G^{\mu\nu}$ by

$$\int d^4y \, K^{\mu\rho}(x-y)G_{\rho\nu}(y-z) = g^\mu_\nu \delta^4(x-z)$$

with

$$K^{\mu\nu}(x-y) \equiv \delta^4(x-y)K^{\mu\nu}(x) \, ,$$

show that

$$G^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \, e^{-ik\cdot(x-y)} \left[-g^{\mu\nu} + (1-\xi)\frac{k^\mu k^\nu}{k^2}\right]\frac{1}{k^2} \, .$$

N.B. If $\xi = 1$ it is the Feynman gauge, while if ξ is chosen to be zero it becomes the Landau gauge.

PROBLEM 2: Interactions of Gauges Bosons with Fermions and Higgs Boson

The covariant derivative of the Standard Model of electroweak interactions is

$$\mathcal{D}_\mu = \partial_\mu + igW_\mu^a t^a + ig' B_\mu \frac{Y}{2}\,, \quad a = 1,2,3,$$

where t^a and $Y/2$ are generators of $SU(2)_L \times U(1)_Y$, $e = g\sin\theta_W = g'\cos\theta_W$, and the charge operator is $Q = t^3 + Y/2$.

Let us consider

$$\mathcal{L} = \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + (D_\mu\Phi)^\dagger (D^\mu\Phi)$$

where $a = 1,2,3$,

$$\psi_{L,R} = \frac{1}{2}\left(1 \mp \gamma_5\right)\psi\,,$$

and

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \quad \text{and } \Phi = \begin{pmatrix} 0 \\ \frac{H+v}{\sqrt{2}} \end{pmatrix},$$

in the unitary gauge with propagator of Z boson to be

$$\Delta_F^{\mu\nu} = \frac{i\left[-g^{\mu\nu} + (k^\mu k^\nu / M_Z^2)\right]}{k^2 - M_Z^2 + i\epsilon}\,, \quad \epsilon \to 0+\ .$$

(a) Derive the relevant interaction Lagrangian involving W_μ^3 and B_μ for $u\bar{u} \to ZH$ with interactions of $Zu\bar{u}$ and HZZ.

(b) Draw Feynman diagrams and describe Feynman rules for $Zu\bar{u}$ and HZZ.

PROBLEM 3: Matrix Element of $u\bar{u} \to ZH$

Draw Feynman diagrams and evaluate the matrix element squared $\langle |M|^2 \rangle$ for $u(p_1)\bar{u}(p_2) \to Z(p_3)H(p_4)$, summed over all spins and polarizations as well as averaged over initial spins and colors.