PHYS 3803—Quantum Mechanics I

Problem Set 1 – Due February 03, 2021

1). A block of mass m moves on a frictionless surface with a displacement x under the influence of a spring force

$$F = -kx,$$

where k is the spring constant.

(a) Calculate the potential energy $V(x)$, and find the Lagrangian.

(b) Derive the Euler-Lagrange equation.

(c) Find the Hamiltonian.

(d) Derive the Hamilton's equations.

2). A particle of mass m moves on a plane under the influence of a central force

$$\vec{F} = -\frac{\alpha}{r^2}\hat{r}, \quad \alpha > 0.$$

where $\hat{r}$ is the unit vector in the radial direction.

(a) Calculate the potential energy $V(r)$, and find the Lagrangian in terms of polar coordinates r and θ.

(b) Derive the Euler-Lagrange equations, then identify the cyclic variables and conserved quantities.

(c) Find the Hamiltonian.

(d) Derive the Hamilton's equations.

3). The action of a mechanical system with N generalized coordinates is defined as

$$S[q_i] = \int L(q_i, \dot{q}_i) dt$$

where $i = 1, \cdots, N$. The Lagrangian L is defined as

$$L(q_i, \dot{q}_i) = T(q_i, \dot{q}_i) - U(q_i)$$

where $T =$ the kinetic energy and $U =$ the potential energy.
Apply the Hamilton's Principle to show that the Euler-Lagrange equations have the following form

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0,$$

for $i = 1, \cdots, N$.

4) **Poisson Brackets.** Consider any continuous functions of the generalized coordinates and momenta $\omega(q_i, p_i)$, $\omega_1(q_i, p_i)$ and $\omega_2(q_i, p_i)$. The Poisson brackets are defined as

$$\{\omega_1, \omega_2\} \equiv \sum_i \left(\frac{\partial \omega_1}{\partial q_i}\frac{\partial \omega_2}{\partial p_i} - \frac{\partial \omega_1}{\partial p_i}\frac{\partial \omega_2}{\partial q_i}\right).$$

Verify the following properties of the Poisson brackets.

(a). $d\omega/dt = \{\omega, H\} + \partial\omega/\partial t$,

(b). $\dot{q}_i = \{q_i, H\}$ and $\dot{p}_i = \{p_i, H\}$,

(c). $\{q_i, q_j\} = 0$ and $\{p_i, p_j\} = 0$,

(d). $\{q_i, p_j\} = \delta_{ij}$,

where H is the Hamiltonian. If the Poisson bracket of two quantities vanishes, the quantities are said to commute. If the Poisson bracket of two quantities equals unity, the quantities are canonically conjugate to each other. Poisson-bracket formalism is very important in Quantum Mechanics.