PHYS 5393: Quantum Mechanics I
Problem Set 11–Due November 20, 2019

Problem (1)

Let us consider the operator

$$M_i = \frac{1}{2m}\epsilon_{ijk}(P_j L_k - L_j P_k) - \frac{e^2}{R} X_i, \quad i,j,k = 1,2,3,$$

where ϵ_{ijk} is the anti-symmetric Levi-Civita symbol,

$$R = \sqrt{X_\ell X_\ell}$$

and the Hamiltonian for the Hydrogen atom is

$$H = \frac{P^2}{2m} - \frac{e^2}{R}.$$

Show that

(a) $[M_i, H] = 0$,

(b) $[M_i, L_j] = i\hbar\epsilon_{ijk} M_k$, and

(c) $[M_i, M_j] = \frac{-2i\hbar}{m}\epsilon_{ijk} H L_k$.

N.B. In tensor notation, repeated indices imply summation.

Problem (2)

Consider two harmonic oscillators with raising and lowering operators $a^\dagger, a$ for the first oscillator and $b^\dagger, b$ for the second, with the following commutation relations

$$[a, a^\dagger] = 1, \quad \text{and} \quad [b, b^\dagger] = 1.$$

All other commutators vanish. From the four products $a^\dagger b, b^\dagger a, a^\dagger a, b^\dagger b$, show that by linear combinations, we can find operators which have the same commutation relations as the angular momentum operators. What operator plays the role of angular momentum squared?