Problems (1): Yukawa Interaction

The interaction between a Dirac field $\psi(x)$ and a charge neutral Klein-Gordon field $\phi(x)$ can be described with the following Lagrangian density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$

$$\mathcal{L}_0 = \bar{\psi}(i \gamma \partial - m)\psi + \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - \frac{1}{2} M^2 \phi^2$$

$$\mathcal{L}_I = -g \bar{\psi}\psi\phi - \frac{\lambda}{4!} \phi^4.$$  

Let us consider $p_1$ and $p_3$ to be momenta for fermions, $p_2$ and $p_4$ as momenta for anti-fermions, and $k$ as the momentum for a scalar. (a) Evaluate the matrix element

$$M_1 = \langle 0 \mid S_1^{(1)} \mid p_1, r; p_2, s; k \rangle$$

with

$$S_1^{(1)} = -ig \int d^4x \left[ \bar{\psi}^{(+)}(x)\psi^{(+)}(x)\phi^{(+)}(x) \right].$$

(b) Evaluate the matrix element

$$M_2 = \langle p_3, s_3; p_4, s_4 \mid S_2^{(2)} \mid p_1, s_1; p_2, s_2 \rangle$$

with

$$S_2^{(2)} = -\frac{g^2}{2} \int d^4x \int d^4y \left[ i D_F(x - y) \right] : \bar{\psi}(x)\psi(x)\bar{\psi}(y)\psi(y) : .$$

Problems (2): Mass Renormalization

Evaluate the lowest order correction to the propagator (FIG 1) for the $\phi^4$ theory in the $N-$dimensional momentum space with $N = 4 - 2\epsilon$. Separate the divergent and finite parts and express the divergent part in terms of $1/\epsilon$.

![Figure 1: One-loop diagram at the lowest order.](image)