PHYS 6213: Advanced Particle Physics, Spring 2022

Lecture 22, Apr 06, 2022 (Wednesday)

- Reading:
  (a) Chap 12 in Collider Physics
  (b) Chap 28 in Quantum Field Theory
- Assignments:
  (a) Problem Set 4 due Apr 08 (Fri)
- TMVA Tutorial on Apr 08 (Fri) 03:00 PM–04:00 PM (On Zoom)

Topics for Today:

Chapter 11 The Higgs Mechanism

11.3 The Higgs Potential

*11.4 Massive Gauge Bosons

11.5 Quarks and Leptons

11.6 The CKM Matrix

Topics for Next Lecture:

12.1 Dimensional Regularization

12.2 Renormalization in the $\lambda\phi^4$ Theory

11.3 The Higgs Potential

Let us consider a scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with complex fields ϕ^+ and ϕ^0. The quantum numbers of the spin-0 fields are

	t	t^3	Q	$Y/2$
ϕ^+	$1/2$	$+1/2$	1	$1/2$
ϕ^0	$1/2$	$-1/2$	0	$1/2$

where $Q = t^3 + Y/2$,

- The Higgs Doublet: Φ
- The Higgs Field: H
- The Goldstone Boson Fields: $G^\pm, G^0$

In the Higgs basis, the Higgs doublet becomes

$$\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\
&= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \langle\Phi\rangle_0 \qquad (1) \\
\langle\Phi\rangle_0 &= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad (2)
\end{aligned}$$

where

- $\phi^+ = G^+$.
- $\phi^0 = (H + iG^0)/\sqrt{2}$.

The Lagrangian involving Higgs doublet is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{KE} - V[\Phi^\dagger\Phi] - V[\phi,\bar{\psi},\psi] \\ &= (D_\mu\Phi)^\dagger(D^\mu\Phi) + \bar{\psi}[i\,\not{D} - m]\psi - V[\Phi^\dagger\Phi] - V[\phi,\bar{\psi},\psi]\,. \end{aligned} \tag{3}$$

The Kinetic Lagrangian

The kinetic energy Lagrangian of the spin-0 bosons

$$\mathcal{L}_{KE} = -\frac{1}{2}(\partial_\mu H \partial^\mu H + m_H^2 H^2) - \partial_\mu G^+ \partial^\mu G^- - \frac{1}{2}\partial_\mu G^0 \partial^\mu G^0 \,. \qquad (4)$$

The kinetic energy Lagrangian of the spin-$\frac{1}{2}$ fermions

$$\begin{aligned} \mathcal{L}_{KE} &= \bar{\psi}[i \not{D} - m]\psi \\ &= \bar{\psi}[i \not{\partial} - m]\psi - g_i \bar{\psi} \not{A}_i^a T_i^a \psi \quad i = 1, 2, 3 \,. \end{aligned} \qquad (5)$$

The Interactions among spin-0 fields

The Interactions among the spin-0 bosons

$$\begin{aligned} \mathcal{L}[H, G] &= -\frac{1}{8}\frac{M_H^2}{v^2}(H^2 + {G^0}^2 + 2G^+G^-)^2 \\ &\quad -\frac{1}{2}\frac{M_H^2}{v} H(H^2 + {G^0}^2 + 2G^+G^-) + \text{quartic terms} \,. \end{aligned} \qquad (6)$$

The Lagrangian density for the scalar doublet is

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \; .$$

The most general renormalizable form of the potential is

$$\begin{aligned} V(\Phi) &= \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \\ &= \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \; . \end{aligned}$$

Let's consider

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \; ,$$

and apply the minimization condition

$$0 = \frac{\partial V(|\Phi|)}{\partial |\Phi|} = 2\mu |\Phi| + 4\lambda |\Phi|^3 \; .$$

The minimum occurs at

$$|\Phi_0| = \left(-\frac{\mu^2}{2\lambda} \right)^{1/2} \; .$$

Quantum analog of a classical minimum is the vacuum expectation value (VEV) of a scalar field,

$$\langle \Phi \rangle_0 = \left(-\frac{\mu^2}{2\lambda} \right)^{1/2} \equiv \frac{v}{\sqrt{2}} \ \text{ for } \ \mu^2 < 0 \ .$$

For simplicity, let us adopt the unitary gauge and define define

$$\Phi = \begin{pmatrix} 0 \\ \frac{H(x)+v}{\sqrt{2}} \end{pmatrix} \tag{7}$$

where H is a real field with a zero VEV.

Applying the minimization condition, we obtain the Higgs potential

$$
\begin{aligned}
V(H) &= \frac{1}{2}\mu^2(H+v)^2 + \frac{\lambda}{4}(H+v)^4 \\
&= \frac{1}{2}\mu^2 H^2 + \mu^2 vH + v + \frac{1}{2}\mu^2 v^2 \\
&\quad + \frac{\lambda}{4}H^4 + \frac{\lambda}{v}H^3 + \frac{6\lambda}{4}v^2 H^2 + \frac{\lambda}{v}^3 H + \frac{\lambda}{4}v^4 \\
&= \frac{1}{2}\mu^2 H^2 + \mu^2 vH + v + \frac{1}{2}\mu^2 v^2 \\
&\quad - \frac{\mu^2}{4v^2}H^4 - \frac{\mu^2}{v}H^3 - \frac{6\mu^2}{4}H^2 - \mu^2 vH - \frac{\mu^2 v^4}{4} \\
&= -\mu^2 H^2 + \frac{1}{4}(\mu^2 v^2)\left(1 - 4\frac{H^3}{v^3} - \frac{H^4}{v^4}\right) \\
&= \frac{1}{2}M_H^2 H^2 + \frac{1}{2}\frac{M_H^2}{v}H^3 + \frac{M_H^2}{8v^2}H^4 \,.
\end{aligned}
$$

And the Higgs Lagrangian becomes

$$\begin{aligned}\mathcal{L}_\phi &= \frac{1}{2}\partial_\mu H \partial^\mu H + \mu^2 H^2 - \frac{1}{4}(\mu^2 v^2)\left(1 - 4\frac{H^3}{v^3} - \frac{H^4}{v^4}\right) \\ &= \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 - \frac{1}{2}\frac{M_H^2}{v} H^3 - \frac{M_H^2}{8v^2} H^4 \,.\end{aligned}$$

The covariant derivative is

$$
\begin{aligned}
D_\mu &= \partial_\mu + igW_\mu^a t^a + ig' B_\mu \left(\frac{Y}{2}\right) \\
&= \partial_\mu + ig(W_\mu^+ t^+ + W_\mu^- t^-) + ieA_\mu Q + i\frac{g}{\cos\theta_W} Z_\mu (t^3 - \sin^2\theta_W Q) \ .
\end{aligned}
$$

Then

$$
\begin{aligned}
D_\mu \Phi &= \left[\partial_\mu + i\frac{g}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} + i\frac{g}{2\cos\theta_W} \begin{pmatrix} Z_\mu & 0 \\ 0 & Z_\mu \end{pmatrix}\right] \begin{pmatrix} 0 \\ \frac{H+v}{\sqrt{2}} \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{bmatrix} i\frac{g}{\sqrt{2}} W_\mu^+ (H+v) \\ \partial_\mu H + i\frac{g}{2\cos\theta_W} Z_\mu (H+v) \end{bmatrix} \ .
\end{aligned}
$$

Here are Infinitesimal Gauge Transformations

$$U(x) = 1 - ig\epsilon^a(x)t^a - ig'\epsilon^B(x)\frac{Y}{2}\,, \quad c_W = \cos\theta_W\,, s_W = \sin\theta_W$$

$$\delta = -ig(\epsilon^+ t^+ + \epsilon^- t^-) - i\frac{g}{c_W}\epsilon^Z(t^3 - s_W^2 Q) - igs_W\epsilon^A Q$$

$$\delta W_\mu^a(x) = \partial_\mu\epsilon^a(x) + g\epsilon^{abc}\epsilon^b(x)W_\mu^c(x) \quad \text{and} \quad \delta B_\mu(x) = \partial_\mu\epsilon^B(x)$$

$$\delta W_\mu^+ = \partial_\mu\epsilon^+ + ig\epsilon^+(c_W Z_\mu + s_W A_\mu) - ig(c_W\epsilon^Z + s_W\epsilon^A)W_\mu^+$$

$$\delta Z_\mu = \partial_\mu\epsilon^Z - igc_W(\epsilon^+ W_\mu^- - \epsilon^- W_\mu^+)$$

$$\delta A_\mu = \partial_\mu\epsilon^A - igs_W(\epsilon^+ W_\mu^- - \epsilon^- W_\mu^+)\,, \quad g\sin\theta_W = e$$

$$\delta G^+ = -i[\frac{1}{2}g\epsilon^+ H + M_W\epsilon^+ + \frac{1}{2}g(\frac{(1-2s_W^2)}{c_W}\epsilon^Z + 2s_W\epsilon^A)G^+] + \frac{1}{2}g\epsilon^+ G^0$$

$$\delta G^0 = -\frac{1}{2}g(\epsilon^+ G^- + \epsilon^- G^+) + \frac{1}{2}\frac{g}{c_W}(\epsilon^Z H) + M_Z\epsilon^Z$$

$$\delta H = -i\frac{1}{2}g(\epsilon^+ G^- - \epsilon^- G^+) - \frac{1}{2}\frac{g}{c_W}(\epsilon^Z G^0)\,.$$

11.5 The Yukawa Couplings of the Quarks

Let's define

$$Q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{8}$$

To generate masses for the quarks we need

$$\begin{aligned} \Phi &= \begin{pmatrix} G^+ \\ \frac{v+H+iG^0}{\sqrt{2}} \end{pmatrix}, \\ \langle \Phi \rangle &= \frac{v}{\sqrt{2}} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \tilde{\Phi} &= i\sigma_2 \Phi^* \\ &= \begin{pmatrix} \frac{v+H+iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix} \end{aligned} \qquad (10)$$

where $G^{\pm}$ and G^0 are Goldstone bosons.

The Yukawa Interactions of the quarks can be written as

$$\begin{aligned} \mathcal{L}_{\phi\bar{q}q} &= -[\quad \tilde{G}_{ab}\bar{Q}_L^a \tilde{\Phi} u_R^b + G_{ab}\bar{Q}_L^a \Phi d_R^b + \tilde{G}_{ab}^* u_R^a \tilde{\Phi}^\dagger Q_L^b + G_{ab}^* \bar{d}_R^a \Phi^\dagger Q_L^b] \\ &= -[\quad \tilde{G}_{ab}(\bar{u}^a, \bar{d}^a)_L \begin{pmatrix} \frac{v+H+iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix} u_R^b \\ &\qquad + G_{ab}(\bar{u}^a, \bar{d}^a)_L \begin{pmatrix} G^+ \\ \frac{v+H+iG^0}{\sqrt{2}} \end{pmatrix} d_R^b + H.C.] \end{aligned} \qquad (11)$$

In the weak eigenstates the quarks are u^a and d^a, $a = 1, 2, 3$.

$$\begin{aligned}\mathcal{L}_{\phi\bar{q}q} \quad = \quad & -\frac{\tilde{G}_{ab}}{\sqrt{2}}[\bar{u}^a_L u^b_R(v + H - iG^0) - \sqrt{2}\bar{d}^a_L u^b_R G^-] \\ & -\frac{G_{ab}}{\sqrt{2}}[\sqrt{2}\bar{u}^a_L d^b_R G^+ + \bar{d}^a_L d^b_R(v + H + iG^0)] + H.C. \end{aligned} \tag{12}$$

Introducing unitary transformations

$$\begin{pmatrix} u^1 \\ u^2 \\ u^3 \end{pmatrix} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \tag{13}$$

and

$$\begin{pmatrix} d^1 \\ d^2 \\ d^3 \end{pmatrix} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} \tag{14}$$

such that

$$U_R^\dagger \tilde{G}_{ab}^* \frac{|v|}{\sqrt{2}} U_L = U_L^\dagger \tilde{G}_{ab} \frac{|v|}{\sqrt{2}} U_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{15}$$

and

$$D_R^\dagger G_{ab}^* \frac{|v|}{\sqrt{2}} D_L = D_L^\dagger G_{ab} \frac{|v|}{\sqrt{2}} D_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \tag{16}$$

$$
\begin{aligned}
\mathcal{L}_{\phi\bar{q}q} \quad = -[\quad & +m_u \bar{u}u + m_d \bar{d}d + \frac{m_u}{v}\bar{u}uH + \frac{m_d}{v}\bar{d}dH \\
& +i\frac{m_u}{v}\bar{u}\gamma_5 uG^0 - i\frac{m_d}{v}\bar{d}\gamma_5 dG^0 \\
& -\frac{1}{\sqrt{2}}\frac{m_u}{v}\bar{u}V_{ud}(1+\gamma_5)dG^+ - \frac{1}{\sqrt{2}}\frac{m_u}{v}\bar{d}V_{ud}(1-\gamma_5)uG^- \\
& +\frac{1}{\sqrt{2}}\frac{m_d}{v}\bar{u}V_{ud}(1-\gamma_5)dG^+ + \frac{1}{\sqrt{2}}\frac{m_d}{v}\bar{d}V_{ud}(1+\gamma_5)uG\,]
\end{aligned}
\tag{17}
$$