CHAPTER I

INTRODUCTION

In all branches of physical science and engineering, one deals constantly with numbers which result more or less directly from experimental observations. In fact, it can be said that the very essence of physical science is the discovering and the using of correlations among quantitative observations of physical phenomena.

Experimental observations always have inaccuracies. In using numbers which result from experimental observations, it is almost always necessary to know the extent of these inaccuracies. If several observations are used to compute a result, one must know how the inaccuracies of the individual observations contribute to the inaccuracy of the result. If one is comparing a number based on a theoretical prediction with one based on experiment, it is necessary to know something about the accuracies of both of these if one is to say anything intelligent about whether or not they agree. If one has some knowledge of the statistical behavior of errors of observation, it is often possible to reduce the effect of these uncertainties on the final result. Such problems as these will be discussed in the following pages.

1 | Kinds of Errors

In discussing errors in individual observations, it is customary to distinguish between *systematic* errors and *chance* or *random* errors.

Systematic errors are errors associated with the particular instruments or technique of measurement being used. Suppose we have a book which is 9 in. high. We measure its height by laying a ruler against it, with one end of the ruler at the top end of the book. If the first inch of the ruler has been previously cut off, then the ruler is likely to tell us that the book is 10 in. long. This is a *systematic* error. If a thermometer immersed in boiling pure water at normal pressure reads 102°C, it is improperly calibrated. If readings from this thermometer are incorporated into experimental results, a systematic error results. An ammeter which is not properly "zeroed" introduces a systematic error.

Very often, in experimental work, systematic errors are more important than chance errors. They are also, however, much more difficult to deal with. There are no general principles for avoiding systematic errors; only an experimenter whose skill has come through long experience can consistently detect systematic errors and prevent or correct them.

Random errors are produced by a large number of unpredictable and unknown variations in the experimental situation. They can result from small errors in

judgment on the part of the observer, such as in estimating tenths of the smallest scale division. Other causes are unpredictable fluctuations in conditions, such as temperature, illumination, line voltage, or any kind of mechanical vibrations of the equipment. It is found empirically that such random errors are frequently distributed according to a simple law. This makes it possible to use statistical methods to deal with random errors. This statistical treatment will form the principal body of the following discussion.

There is a third class, containing what are sometimes called errors but which are not, properly speaking, errors at all. These include mistakes in recording numbers, blunders of reading instruments incorrectly, and mistakes in arithmetic. These types of inaccuracies have no place in a well-done experiment. They can always be eliminated completely by careful work.

The terms *accuracy* and *precision* are often used to distinguish between systematic and random errors. If a measurement has small *systematic* errors, we say that it has high *accuracy;* if small *random* errors, we say it has high *precision.*

2 | Propagation of Errors

Propagation of errors is nothing but a fancy way of describing the obvious fact that if one uses various experimental observations to calculate a result, and if the observations have errors associated with them, then the result will also be in error by an amount which depends on the errors of the individual observations.

Ordinarily it is not possible to calculate directly the errors in the results, because the errors in the observations are not usually known. If we knew them, we could correct the observations and eliminate the errors! The results of this section are thus not directly useful for treating propagation of experimental errors, but they can be used to obtain formulas which are useful. This will be the principal task of Sec. 13. Meanwhile, the results obtained in this section are directly useful in cases where the "error" is not really an error but a small change in the value of a known quantity, and we want to compute the effect which this change has on the result of a calculation which contains this quantity.

For example, suppose one wants to determine the volume of a cylinder by measuring its radius r and its height h, using the formula

$$V = \pi r^2 h \tag{2.1}$$

There may be an error in the measurement of r, so that the result of our measurement is not r but something slightly different, say $r + \Delta r$ (where Δr is the error). If there is a similar error Δh in measuring the height, then our result is not V, the true value, but something slightly different, $V + \Delta V$. We can calculate ΔV as follows. In the formula we place $r + \Delta r$ instead of just r and $h + \Delta h$ instead of h; then the result is $V + \Delta V$:

$$V + \Delta V = \pi(r + \Delta r)^2(h + \Delta h) \tag{2.2}$$

If we expand this, and subtract V from both sides of the equation, the result is

$$\Delta V = \pi(r^2\,\Delta h + 2rh\,\Delta r + \Delta r^2 h + 2r\,\Delta r\,\Delta h + \Delta r^2\,\Delta h) \tag{2.3}$$

Now if the error Δr is much smaller than r itself, and if

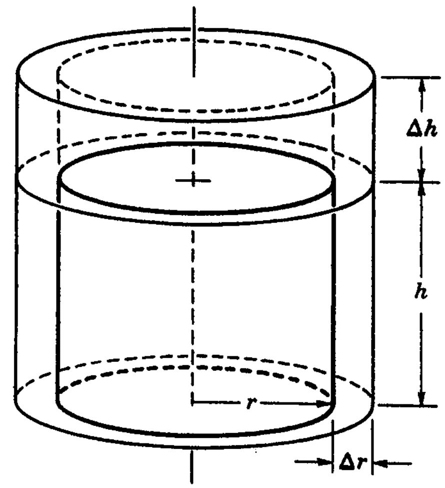


Fig. 2.1. Changes in the volume of a cylinder resulting from changes Δr and Δh in its dimensions. Can you identify the separate terms of Eq. (2.4) in the figure?

Δh is much smaller than h, the last three terms in Eq. (2.3) are much smaller than the first two; hence we can write approximately

$$\Delta V \cong \pi(r^2\,\Delta h + 2rh\,\Delta r) \tag{2.4}$$

which allows us to calculate the error ΔV if we know r, h, and their errors. Describing this result in different words, we may say that Eq. (2.4) gives a means of calculating how much the volume of a cylinder changes if we change its dimensions by the amounts Δr and Δh.

Often we are interested not in the error itself, but in the so-called fractional error, which is defined as the *ratio* of the error of the quantity to the true value of the quantity; in the present case this is $\Delta V/V$. Using Eqs. (2.1) and (2.4), we obtain

$$\frac{\Delta V}{V} \cong \frac{\pi(r^2\,\Delta h + 2rh\,\Delta r)}{\pi r^2 h} = \frac{\Delta h}{h} + \frac{2\Delta r}{r} \tag{2.5}$$

This is a remarkably simple result because it shows that the fractional error in V is related very simply to the fractional errors (or fractional changes) of the quantities h and r which are used to determine V.

This same result can be obtained in a slightly different way. We can approximate the error in V resulting from the error in r by means of derivatives. If the errors ΔV and Δr are small, then the ratio $\Delta V/\Delta r$ is approximately equal to the derivative dV/dr. But $dV/dr = 2\pi rh$. Hence, we have approximately

$$\frac{\Delta V}{\Delta r} \cong 2\pi rh \quad \text{and} \quad \frac{\Delta V}{V} \cong \frac{2\pi rh\,\Delta r}{\pi r^2 h} = 2\frac{\Delta r}{r} \tag{2.6}$$

This gives the part of the fractional error in V which results from the error in r. A similar calculation gives the contribution of Δh, and the total fractional error $\Delta V/V$ is the same as obtained previously.

Because V is a function of both r and h, the correct mathematical language for the derivative of V with

respect to r which we have used above is $\partial V/\partial r$, which is read "partial derivative of V with respect to r," and means simply that we recognize that V is a function of other variables besides r, but we are evaluating the derivative with respect to r, while all the other variables are kept constant. Similarly, we can define a partial derivative of V with respect to h, $\partial V/\partial h$. An approximate expression for the error ΔV can then be written:

$$\Delta V \cong \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h \tag{2.7}$$

Let us generalize this result. Suppose we have a quantity Q which depends upon several observed quantities $a, b, c, \ldots$. The error ΔQ resulting from errors Δa, $\Delta b, \ldots$ can be represented as

$$\Delta Q = \frac{\partial Q}{\partial a}\Delta a + \frac{\partial Q}{\partial b}\Delta b + \frac{\partial Q}{\partial c}\Delta c + \cdots \tag{2.8}$$

and the fractional error $\Delta Q/Q$ as

$$\frac{\Delta Q}{Q} = \frac{1}{Q}\frac{\partial Q}{\partial a}\Delta a + \frac{1}{Q}\frac{\partial Q}{\partial b}\Delta b + \cdots \tag{2.9}$$

As was mentioned at the beginning of this section, the discussion just given is not of much direct usefulness in the analysis of propagation of errors. We have talked as though we knew the true values of the observed quantities, along with the errors in the observations. In some particular cases this may be true; or we may want to compute the change in Q which results from given values of $a, b, \ldots$. Then we may use Eq. (2.8).

But often this is not the case. Ordinarily we do not know the errors exactly because errors usually occur randomly. Often the *distribution* of errors in a set of observations is known, but the error in any individual observation is not known. Later, after acquiring some fundamental knowledge of statistical methods, we shall learn in Sec. 13 some considerably more sophisticated methods for treating problems in propagation of errors. The methods of Sec. 13 will be of much greater practical usefulness than the naïve considerations given in this section.

Another consideration is that it is not always clear whether or not such a thing as a "true value" really *exists*. Suppose we are trying to measure the length of a broken stick, whose ends are uneven and jagged. We may be able to state that the length is between certain limits, say between 14 and 15 in. But if we try to be more precise we have to decide where the ends are; if we aspire to measure the length to within 0.01 in., we cannot say that to this precision the stick *has* a definite length.

In most of what follows, we shall assume that we are making measurements on quantities for which true values really exist. We should keep in mind, however, that there are areas of physics in which it is not correct to say that a particular observable quantity *has* a definite value. This basic uncertainty of some basic physical quantities is, in fact, one of the fundamental notions of quantum mechanics. In quantum-mechanical problems

one must often be content with the statement that the *average* of a large number of observations has a definite value.

3 | The Mean and Dispersion

Suppose we want to make an accurate measurement of the diameter of a hard steel rod with a micrometer caliper. Assuming that the rod *has* a "true diameter," we will probably get several different results if we make the measurement several times. We may tighten the micrometer more sometimes than others, there may be small dust particles present, we may make small errors in estimating tenths of the smallest scale division, etc. Still, one suspects intuitively that it should be possible to obtain a more reliable result for the diameter by using the 10 measurements than by using only one measurement.

What then shall we do with the 10 measurements? The first procedure which obviously suggests itself is simply to take the *average*, or arithmetic *mean*. The *mean* of a set of numbers is defined as the sum of all the numbers divided by the number of them. If we have 10 measurements we add them all up and divide by 10. In a more general language which we shall use often, let us call a typical observation x_i. If there are 10 observations, then the index i can have any value from 1 to 10. If there are N observations, then i ranges from 1 to N.

In general, we may define the mean $\bar{x}$ of the set of numbers x_i as

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_{N-1} + x_N}{N} \tag{3.1}$$

In what follows, a bar over a letter will always signify a "mean value." A convenient mathematical shorthand which we frequently use is

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{3.2}$$

In this expression, the symbol

$$\sum_{i=1}^{N}$$

is read "the sum from $i = 1$ to N"; its meaning is that in the expression which follows Σ we first place $i = 1$, add to it the result of placing $i = 2$ and so on, up to $i = N$, which is the last value of i. Thus,

$$\sum_{i=1}^{N} x_i = x_1 + x_2 + x_3 + \cdots + x_{N-1} + x_N$$

It will be seen later that in some important cases there is a good reason for regarding the *average* of a set of measurements as the *best* estimate of the true value of the quantity being measured. For the present, however, we observe simply that taking the average seems intuitively to be a reasonable procedure.

Sometimes we want to compute the mean of a set of numbers (which may be measurements or anything

else) in which we think that some numbers are more important than others. How shall we make the calculation? If, for example, two observers guess the height of a tree as 30 and 60 ft, respectively, and we have twice as much confidence in the first observer as the second, how shall we compute a combined "best guess" as to the height of the tree?

A procedure which immediately suggests itself is to

Fig. 3.1. Observations with unequal weights.

pretend that the 30-ft guess was made more than once. Suppose, for example, we include it in the average twice. Then, of course, we must divide by the total number of guesses, which is now three. Then our best guess will be

$$\frac{2(30\text{ ft}) + 1(60\text{ ft})}{2 + 1} = 40\text{ ft}$$

More generally, if we have several guesses with different degrees of reliability, we can multiply each by an appropriate weighting factor, and then divide the sum of these products by the sum of all the weighting factors.

Such considerations lead us to the idea of a *weighted mean*. The weighted mean of a set of numbers is defined as follows: For each number x_i in the set $(x_1, x_2, \ldots, x_N)$ we assign a weighting factor, or weight w_i. The weighted mean $\bar{x}$ is then defined as

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \cdots + w_Nx_N}{w_1 + w_2 + \cdots + w_N} = \frac{\sum_{i=1}^{N} w_ix_i}{\sum_{i=1}^{N} w_i} \tag{3.3}$$

Note that if all the weights are unity (or, more generally, if they are all equal) the weighted mean reduces to the mean as previously defined by Eq. (3.2).

Having obtained a set of measurements x_i and the mean $\bar{x}$, we should like to have a way of stating quantitatively how much the individual measurements are scattered away from the mean. A quantitative description of the *scatter* (or *spread* or *dispersion*) of the measurements will give us some idea of the precision of these measurements.

To obtain such a quantitative description, we first define a deviation d_i for each measurement x_i. The deviation d_i is defined as the difference between any measurement x_i and the mean $\bar{x}$ of the set. That is,

$$d_i = x_i - \bar{x} \tag{3.4}$$

(We could equally well have defined d_i as $\bar{x} - x_i$ instead

of $x_i - \bar{x}$; the definition given here is the conventional one. Some authors refer to the d_i as *residuals* rather than deviations. The two terms are synonymous.)

It should be noted here that it would *not* be correct to call d_i the *error* in measurement x_i, because $\bar{x}$ is not actually the true value of the observed quantity. It can be shown that in many cases, if a very large number of observations is made, $\bar{x}$ *approaches* the true value of the quantity (assuming that there are no systematic errors), and then the deviations d_i *approach* the true errors in the measurements x_i. This is the case, for example, if the errors are distributed according to the Gauss distribution, or "normal error function," to be discussed in Sec. 9.

As a first attempt at a quantitative description of the spread or dispersion of the measurements x_i about the mean, we might consider the average of the deviations. This is

$$\frac{1}{N}\sum_{i=1}^{N} d_i = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x}) \tag{3.5}$$

The right-hand side of Eq. (3.5) is a sum of N terms, each one of which is itself a sum of two terms. The order of adding these terms is immaterial; so we could just as well add all the first terms, then add all the second terms; that is,

$$\frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x}) = \frac{1}{N}\left(\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} \bar{x}\right) \tag{3.6}$$

Now what is the meaning of the second term on the right side of Eq. (3.6)? It is a sum of N terms, but they are all the same. We simply add $\bar{x}$ itself N times. That is,

$$\sum_{i=1}^{N} \bar{x} = N\bar{x}$$

Thus the expression for the average of the residuals boils down to

$$\frac{1}{N}\sum_{i=1}^{N} d_i = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x}) = \frac{1}{N}\sum_{i=1}^{N} x_i - \bar{x} = 0 \tag{3.7}$$

because of Eq. (3.2). The average of the residuals is always zero.

This should not be particularly surprising; some of the observations are larger than the mean, and some are smaller than the mean; so some of the residuals are positive, and some are negative. Because of the way we define the average and the residuals, the *average* of the residuals is *always* zero. This means that the average of the residuals is not very useful as a characterization of the scatter or dispersion.

Perhaps a better idea would be to take the absolute value of each residual and average the absolute values. We thereby obtain what is called the *mean deviation*, denoted by α. That is,

$$\alpha = \frac{1}{N}\sum_{i=1}^{N} |d_i| = \frac{1}{N}\sum_{i=1}^{N} |x_i - \bar{x}| \tag{3.8}$$

This quantity is often referred to as the average deviation; this is a misnomer, as is "mean deviation." It is

not the average deviation but the average of the *absolute values* of the deviations. This quantity is sometimes used to characterize the spread or dispersion of the measurements. For various reasons which will be discussed later, it is not so useful as another one which will be defined next, called *standard deviation*.

In defining the standard deviation, we get around the problem of handling the negative residuals by *squaring* each deviation, thereby obtaining a quantity which is always positive. We then take the *average* of the *squares*, and then take the *square root* of this result. Thus the standard deviation can also be referred to as the *root-mean-square deviation*, in that it is the square root of the mean of the squares of the deviations. The standard deviation is usually symbolized by σ, and its defining equation is

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} d_i^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2} \tag{3.9}$$

The square of the standard deviation σ^2 is called the *variance* of the set of observations. Note that σ always has the same units as the x_i, and that it is always positive.

One might now ask, How is σ related to the precision of the *mean*, $\bar{x}$? Clearly, it is unlikely that $\bar{x}$ is in error by as much as σ if the number of observations is large. It will be shown in Sec. 12 that in many cases the error in $\bar{x}$ is not likely to be greater than $\sigma/N^{1/2}$. Thus, as we should expect, more measurements give a more reliable mean.

We now transform Eq. (3.9), which defines σ, into another form which involves only the observations x_i. This new form will not be particularly useful, except perhaps for machine calculations; but it provides us with an excuse for doing some more manipulations with summation symbols similar to the manipulations used in showing that the average of the deviations is zero.

We square the entire expression, and then multiply out the squared term following Σ:

$$\sigma^2 = \frac{1}{N}\sum (x_i - \bar{x})^2 = \frac{1}{N}\sum (x_i^2 - 2x_i\bar{x} + \bar{x}^2) \quad (3.10)$$

In this expression and those which follow, we drop the limits on the summation symbol in order to save writing. Unless otherwise noted, we assume that the summation runs over the number of measurements, that is, from $i = 1$ to N. Now, as before, we separate the various terms in the sum:

$$\sigma^2 = \frac{1}{N}\sum x_i^2 - \frac{1}{N}\sum 2x_i\bar{x} + \frac{1}{N}\sum \bar{x}^2 \quad (3.11)$$

The second term in Eq. (3.11) is a sum in which every term contains the quantity $2\bar{x}$ as a factor. It is therefore legitimate to factor out $2\bar{x}$ and write this term as

$$-\frac{1}{N}\sum 2x_i\bar{x} = -\frac{1}{N}(2\bar{x})\sum x_i = -2\bar{x}^2 \quad (3.12)$$

where we have used the definition of the mean, Eq. (3.2). Furthermore, the third term of Eq. (3.11) contains a sum of N terms, each of which is just $\bar{x}^2$; so the value

of the term is just $\bar{x}^2$. Therefore the whole expression can be written:

$$\sigma^2 = \frac{1}{N}\sum x_i^2 - \bar{x}^2 = \frac{1}{N}\sum x_i^2 - \left(\frac{1}{N}\sum x_i\right)^2 \quad (3.13)$$

It is important to note that in general the quantities $\Sigma(x_i^2)$ and $(\Sigma x_i)^2$ are not equal; if you do not believe this, try writing out some terms of each of these sums.

The following is an example which illustrates the calculation of the mean, the average absolute deviation, and the standard deviation of a set of observations. Here $N = 6$.

i	x_i, in.	d_i, in.	d_i^2, in.^{2}
1	0.251	0.001	0.000001
2	0.248	−0.002	0.000004
3	0.250	0.000	0.000000
4	0.249	−0.001	0.000001
5	0.250	0.000	0.000000
6	0.252	0.002	0.000004
	$\Sigma x_i = 1.500$ in. $\bar{x} = \frac{1}{6}\Sigma x_i$ $= 0.2500$ in.	$\Sigma \lvert d_i \rvert = 0.006$ in. $\alpha = \frac{1}{6}\Sigma \lvert d_i \rvert$ $= 0.001$ in.	$\Sigma d_i^2 = 0.000010$ in.^{2} $\sigma = \sqrt{\frac{1}{6}\Sigma d_i^2}$ $= 0.0013$ in.

In analogy to the fractional errors defined in Sec. 2, we sometimes use the *fractional standard deviation*, defined as the ratio of the standard deviation to the mean $\sigma/\bar{x}$ or the *per cent standard deviation* $(\sigma/\bar{x}) \times 100\%$. In the previous example, the fractional standard deviation is

0.0013 in./0.250 in. = 0.005, and the per cent standard deviation is 0.5%. Note that the fractional standard deviation is always a pure number (without units) because it is always a ratio of two numbers with the same units.

If a weighted mean of the numbers x_i, with weights w_i, has been computed, then the definitions of the mean deviation and standard deviation should be modified somewhat. We shall postpone until later a detailed discussion of how to calculate the standard deviation of a weighted mean. This discussion will be made easier by use of the concept of standard deviation of the mean introduced in Sec. 12 and the analysis of propagation of errors in Sec. 13. By the time we reach these sections, we shall also have some techniques for assigning weights to numbers, in a few situations of practical importance.

PROBLEMS

1. The numerical value of e, the base of natural logarithms, is approximately

$$e = 2.7182\ 8182\ 8459\ 0452\ 3536$$

An infinite series which can be used to compute this value is

$$e = 1 + 1/1 + 1/1\cdot 2 + 1/1\cdot 2\cdot 3 + 1/1\cdot 2\cdot 3\cdot 4 + \cdots$$

Find the fractional error which results from taking the following:

a. The first three terms of the series.

b. The first five terms.

2. The numerical value of π is approximately

$$\pi = 3.1415\ 9265\ 3589\ 7932\ 3846$$

Find the fractional error in the following approximate values:

a. $22/7$.

b. $355/113$.

3. An inaccurate automobile speedometer reads 65 mph when the true speed is 60 mph, and 90 mph when the true speed is 80 mph. Does the fractional error increase or decrease with increasing speed?

4. In Prob. 3, suppose that the error changes proportionately with the speed. At what speed will there be zero error? Is the result the same if instead the *fractional error* is assumed to change proportionately with speed?

5. A certain automobile engine has pistons 3.000 in. in diameter. By approximately what fraction is the piston displacement increased if the cylinder bore (diameter) is increased to 3.060 in. and oversize pistons are installed?

6. A certain type of paper used for stationery is referred to as "twenty pound" because a ream (500 sheets) of 17- by 22-in. sheets weighs 20 lb. If the sheets are $1/16$ in. oversize in each dimension, how much will a ream weigh?

7. If the mass of a stationary particle is m_0, its apparent mass when moving with velocity v is given by relativity theory as $m = m_0(1 - v^2/c^2)^{-1/2}$, where c is the velocity of light, $c = 3 \times 10^8$ m/sec. By what fraction does the mass of an electron differ from its rest mass if its velocity is:

a. 3×10^4 m/sec?

b. 3×10^7 m/sec?

8. It is often convenient to approximate powers of numbers close to unity by using the binomial theorem. For example,

$$(1.01)^2 = (1 + 0.01)^2 = 1 + 2(0.01) + (0.01)^2$$
$$= 1 + 0.02 + 0.0001$$
$$\cong 1.02$$

The error in this approximation is $0.0001/1.0201 \cong 0.01\%$. Show that, in general, if $\delta \ll 1$, then $(1 + \delta)^n \cong 1 + n\delta$, and that the *error* in this approximation is about $\frac{1}{2}n(n - 1)\delta^2$.

9. Use the results of Prob. 8 to obtain the approximation $(A + \delta)^n \cong A^n + n\delta A^{n-1}$, valid when $\delta \ll A$. What is the fractional error in this approximation?

10. Use the method of Prob. 8 to find approximately the values of:

a. $(1.001)^3$
b. $1/0.998$
c. $\sqrt{1.004}$

11. Two lengths a and b are measured with a meter stick, with a possible error of 0.1 cm in each. The values obtained are

$a = 50.0$ cm $\qquad b = 55.0$ cm

a. What is the maximum error in the quantity $(a + b)$? In $(a - b)$?

b. What is the maximum fractional error in $(a + b)$? In $(a - b)$?

12. In a "tangent galvanometer," the current is proportional to the tangent of the angle of deflection of the galvanometer needle. That is, $I = C \tan \theta$. If the error in measuring θ is known, find the value of θ for which:

a. The error in I is smallest.
b. The fractional error in I is smallest.

13. The acceleration of gravity g can be obtained by measuring the period T of a simple pendulum and its length l, using the relation $T = 2\pi\sqrt{l/g}$. Suppose the period was

observed to be 2 sec, with an error of observation of 0.02 sec, and the length was observed to be 1 m, with an error of observation of 0.01 m.

a. What is the maximum error in g? The minimum error?

b. Which of the errors contributes most to the error in g? Why?

14. The components F_x and F_y of a vector with length F, making an angle θ with the positive x axis in an x-y coordinate system, are given by

$F_x = F \cos \theta \qquad F_y = F \sin \theta$

If an error $\Delta\theta$ is made in the measurement of θ, derive expressions for the errors and fractional errors in F_x and F_y.

15. Approximately what fractional errors might be expected in the following measurements:

a. A distance of 10 cm measured with an ordinary meter stick.

b. A mass of 1 g measured with an analytical balance.

c. A ¼-in. steel rod measured with a good micrometer caliper.

d. A human hair measured with a good micrometer caliper.

e. A voltage of 1.5 volts measured with a meter having a scale 3 in. long with full-scale reading 5 volts.

16. Find the mean, standard deviation, and mean deviation of the following set of numbers:

1, 2, 3, 4, 5

17. For the set of numbers (1, 2, 3, 4, 5, 6) find:

a. The mean.

b. The weighted mean in which the weights are 1, 1, 2, 2, 3, 3, respectively.

c. The weighted mean in which the weights are 1, 2, 3, 3, 2, 1, respectively.

18. Ten measurements of the diameter of a hard steel rod with a micrometer caliper yielded the following data:

Diameter, in.

0.250	0.246
0.252	0.250
0.255	0.248
0.249	0.250
0.248	0.252

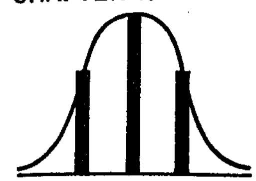
Calculate the standard deviation and mean deviation of this set of measurements.

19. Show that the error in the *mean* of a series of measurements is always smaller than the largest error in an individual measurement.

20. In a certain set of observations, one observation has a much larger deviation from the mean than the others. If this observation is omitted from the calculations, which measure of spread is affected more, the mean deviation or the standard deviation? Why?

21. If the mean of a large set of observations is m, and all deviations between $-e$ and $+e$ occur equally often, find the mean deviation and standard deviation.

CHAPTER II



PROBABILITY

Any quantitative analysis of random errors of observation must be based on probability theory. It is instructive to consider some simple probability calculations first, as preparation for the task of applying probability theory to the study of random errors.

4 | The Meaning of Probability

If we throw a penny up in the air, we know intuitively that the "chance" of its coming down heads is one-half, or 50%. If we roll an ordinary die (singular of dice) we know that the chance of the number 5 coming up is one-sixth.

What does this really mean, though? On each flip of the penny it comes down either heads or tails; there is no such thing as a penny coming down half heads and half tails. What we really mean is that if we flip the penny a very large number of times, the number of times it comes down heads will be approximately one-half the total number of trials. And, if we roll one die