MODELING AN INVERTED PENDULUM

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Computational Physics

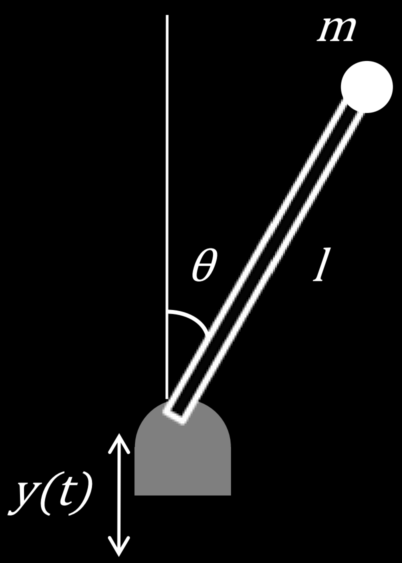
Background Information

- The center of mass of an inverted pendulum is above its point of suspension.
  - If this point is stationary, the pendulum is unstable.
  - If this point is vibrated vertically with a high frequency, the pendulum may be stable.
    - High Frequency ~ 50 Hz
- The pendulum can remain stable despite small disturbances.

Background Information

- This stability is easily demonstrated using:
  - A function generator
  - A PASCO “wave driver” or a speaker
  - A small piece of drinking straw
  - Masking tape
  - A support pin

Background Information



An angular displacement +θ results in a positive torque, which provides a positive angular acceleration:

$$mgl\sin\theta = I\frac{d^2\theta}{dt^2}$$

But if the support point of the pendulum is oscillating vertically with a displacement,

$$y = a\sin\omega t$$

then,

$$ml\sin\theta(g + \omega^2 A\cos\omega t) = I\frac{d^2\theta}{dt^2}$$

Background Information

- The torque,

$$ml \sin\theta (g + \omega^2 A \cos \omega t)$$

averaged over one or more periods of rapid oscillation must become negative in order for the pendulum to be stable.

My Program

- Uses an RK2 integration scheme to solve the differential equation on the previous slide.
- Initial values of θ and d θ/dt are chosen, then updated using:

$$\theta|_{t+\Delta t/2} = \theta|_t + \frac{\Delta t}{2}\frac{d\theta}{dt}\bigg|_t$$

$$\frac{d\theta}{dt}\bigg|_{t+\Delta t/2} = \frac{d\theta}{dt}\bigg|_t + \frac{\Delta t}{2}\frac{d^2\theta}{dt^2}\bigg|_t$$

$$\frac{d^2\theta}{dt^2}\bigg|_{t+\Delta t/2} = \left[g + \omega^2 A \sin\omega\left(t + \frac{\Delta t}{2}\right)\right]\left(\frac{ml}{I}\right)\sin\theta|_{t+\Delta t/2}$$

My Program

- and,

$$\theta|_{t+\Delta t} = \theta|_t + \Delta t \left.\frac{d\theta}{dt}\right|_{t+\Delta t/2}$$

$$\left.\frac{d\theta}{dt}\right|_{t+\Delta t} = \left.\frac{d\theta}{dt}\right|_t + \Delta t \left.\frac{d^2\theta}{dt^2}\right|_{t+\Delta t/2}$$

$$\left.\frac{d^2\theta}{dt^2}\right|_{t+\Delta t} = [g + \omega^2 A \sin \omega(t + \Delta t)] \left(\frac{ml}{I}\right) \sin \theta|_{t+\Delta t/2}$$

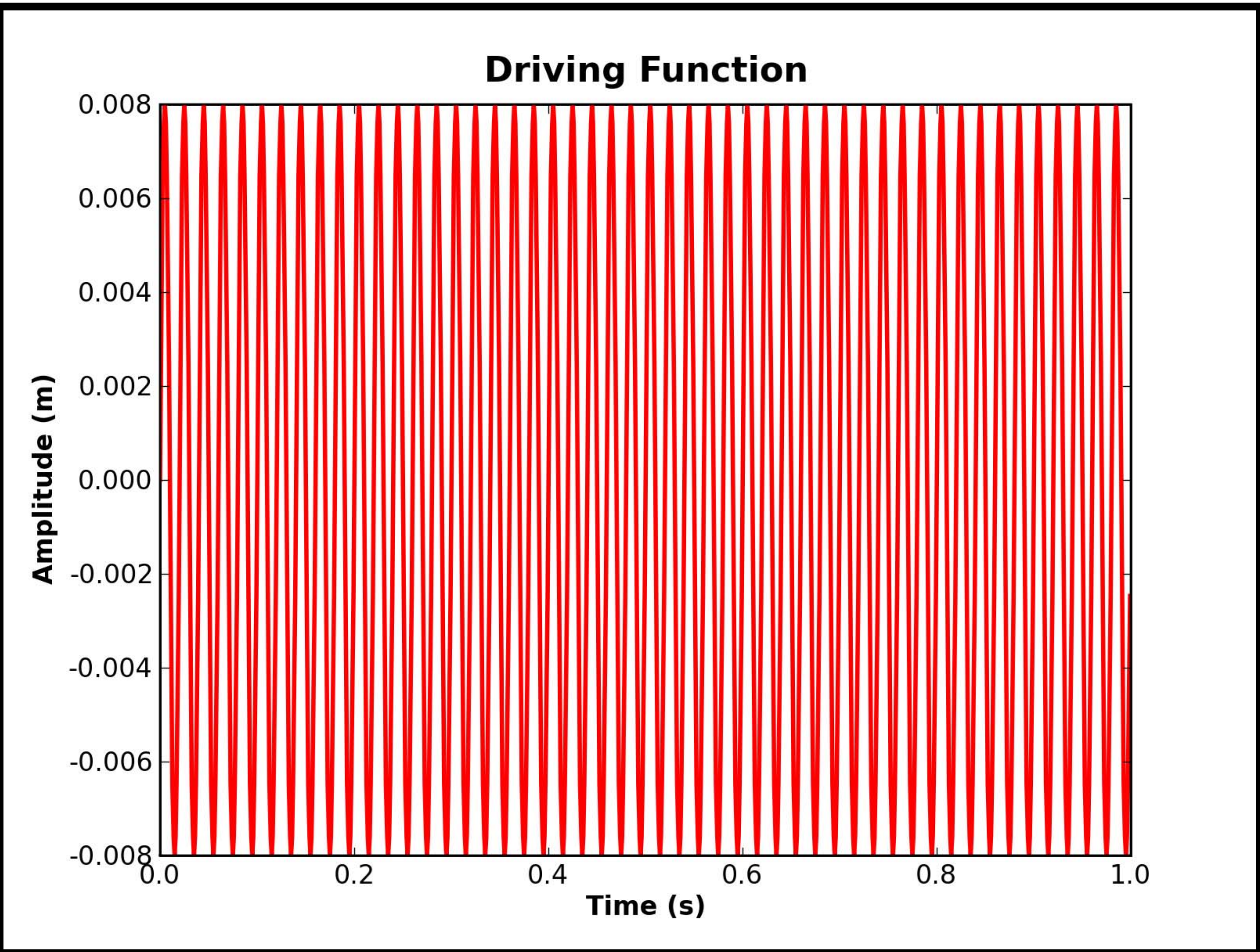
- A time step of 0.001 is used.

My Program

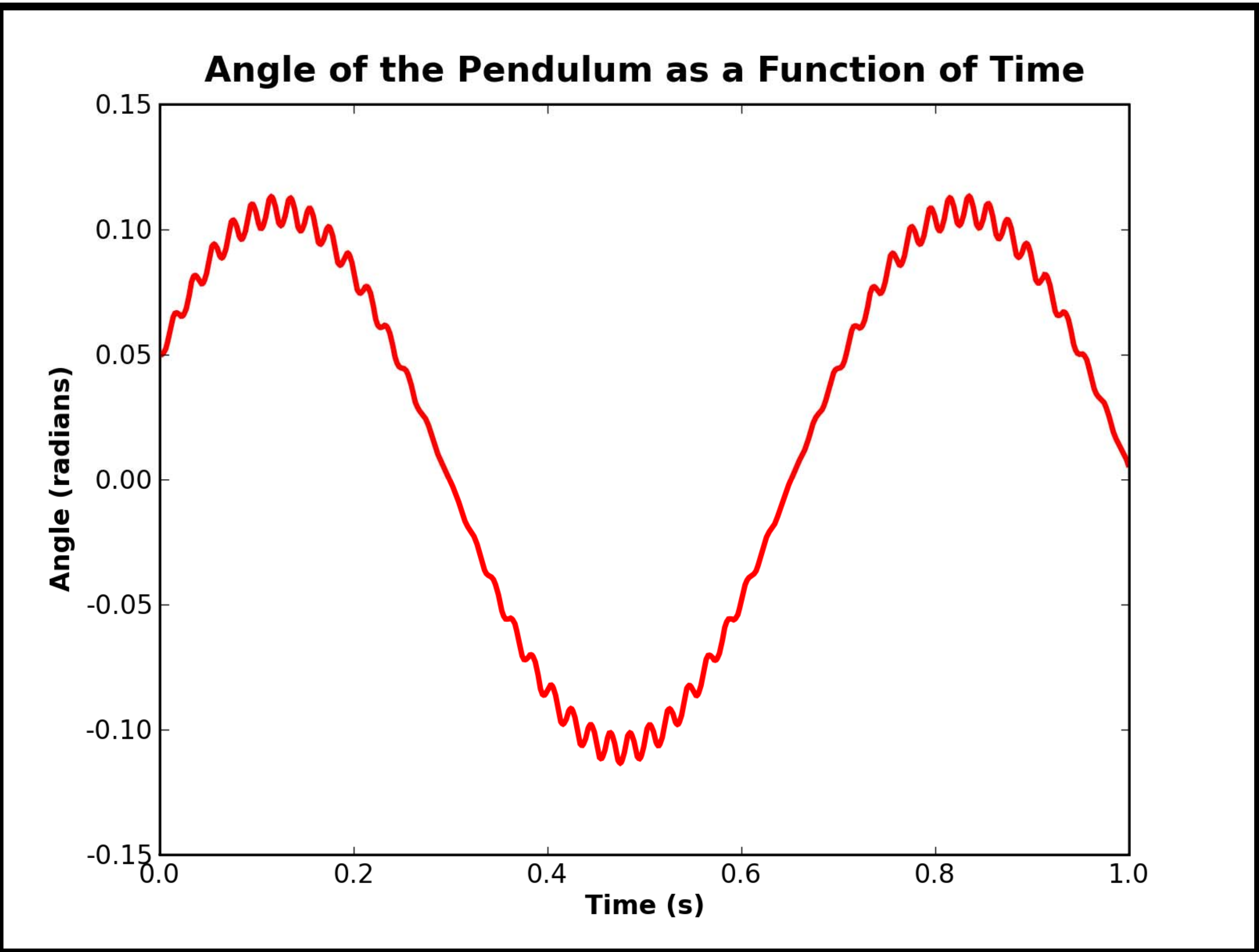
- A simulation was made using the following assumptions:
  - $l = 0.225$m
  - $I = (1/3)m \cdot l^2$
  - $m = 1.0$g
  - Amplitude = 0.8mm
  - Frequency = 50 Hz or 30 Hz

- My simulations shows that the inverted pendulum remains stable at 50 Hz, but not at 30 Hz.

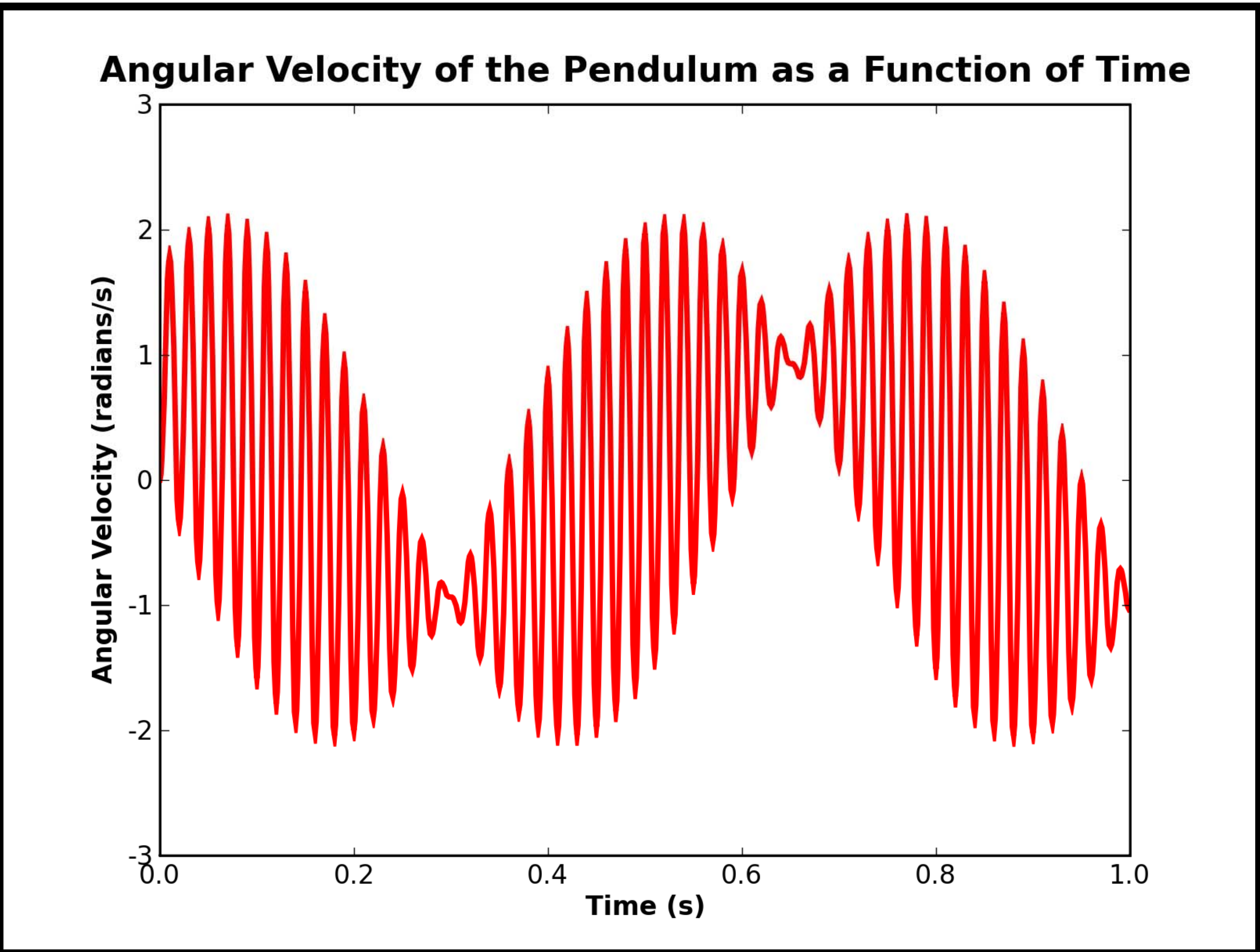
Results – 50Hz



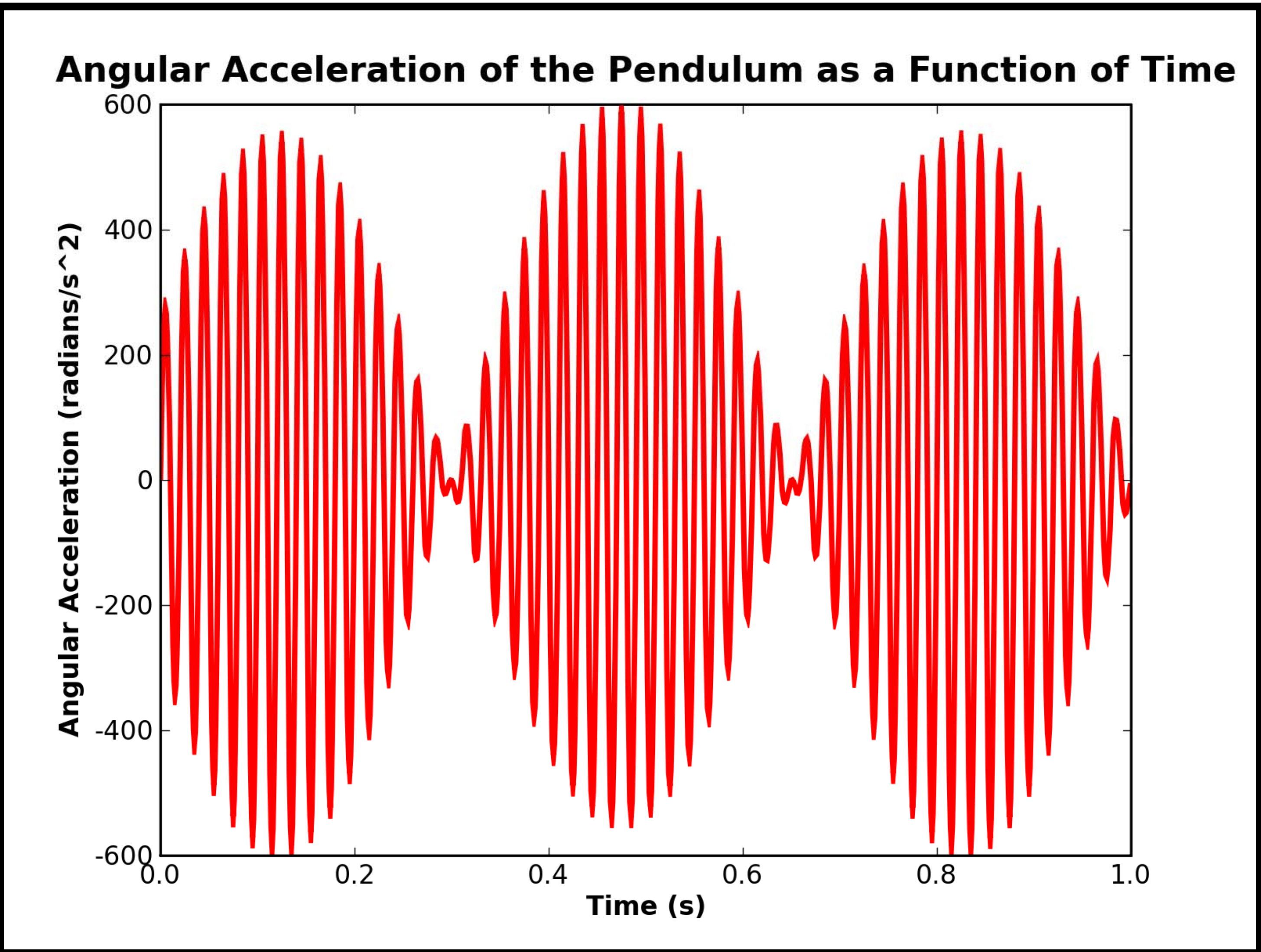
Results – 50Hz



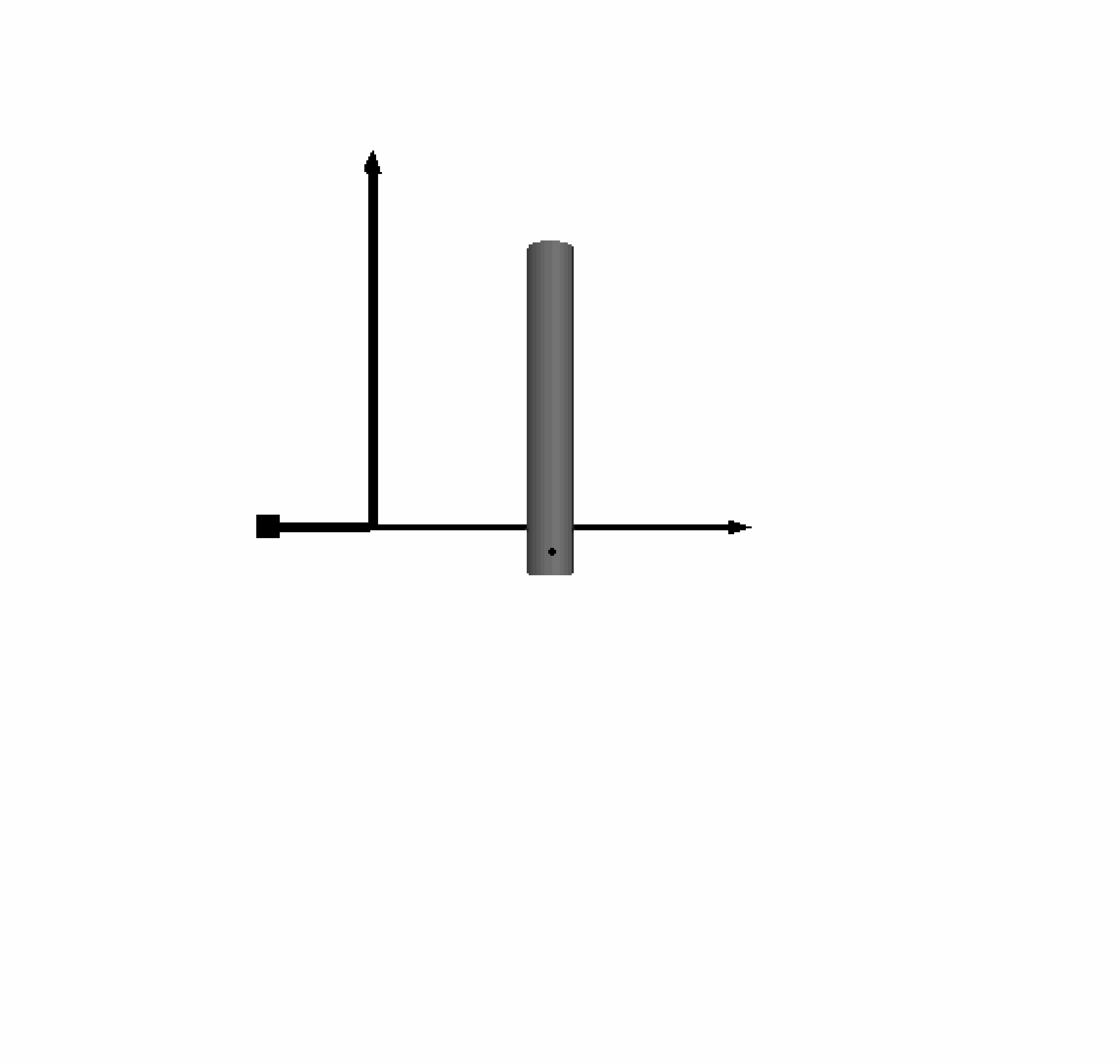
Results – 50Hz



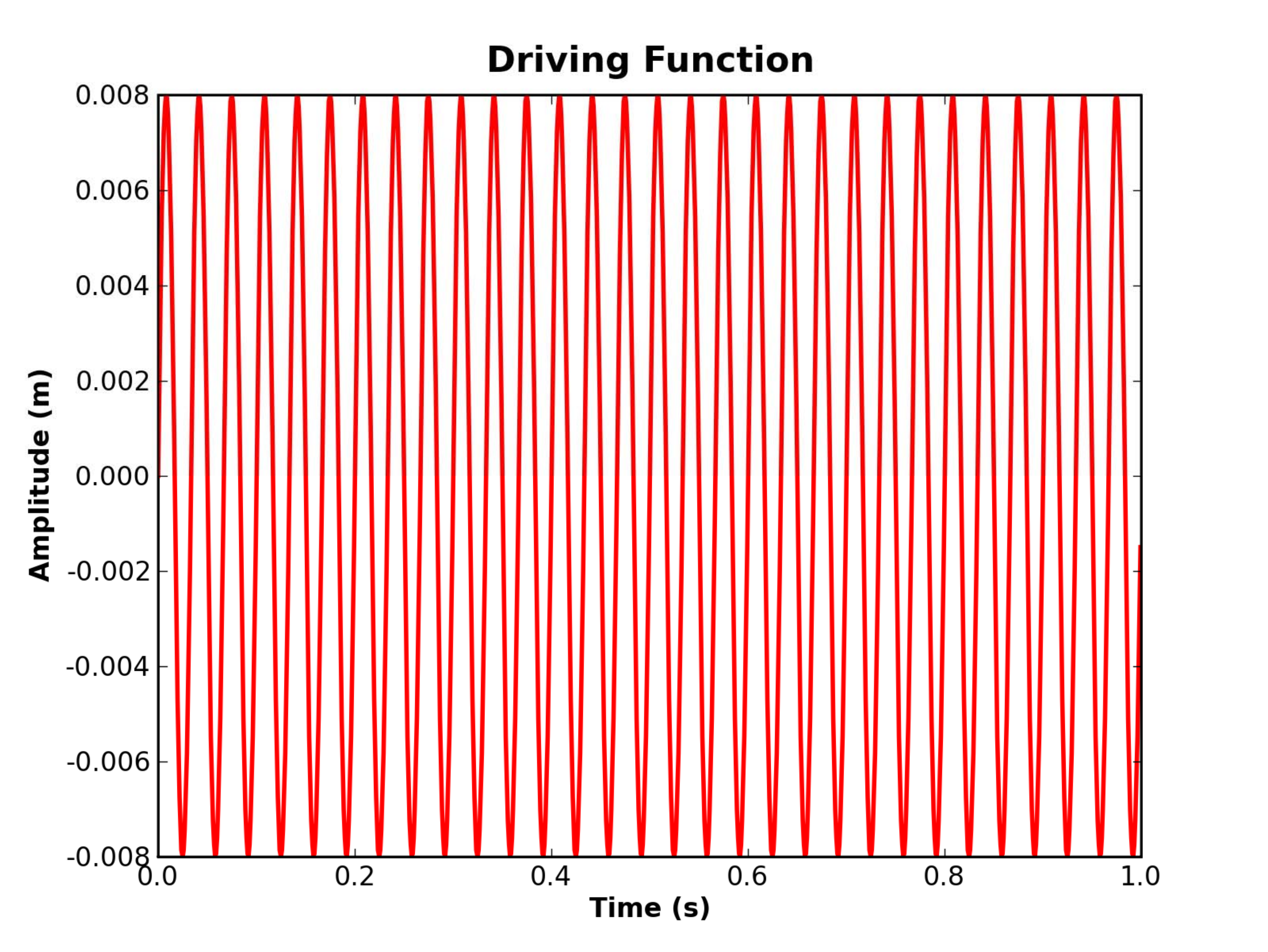
Results – 50Hz



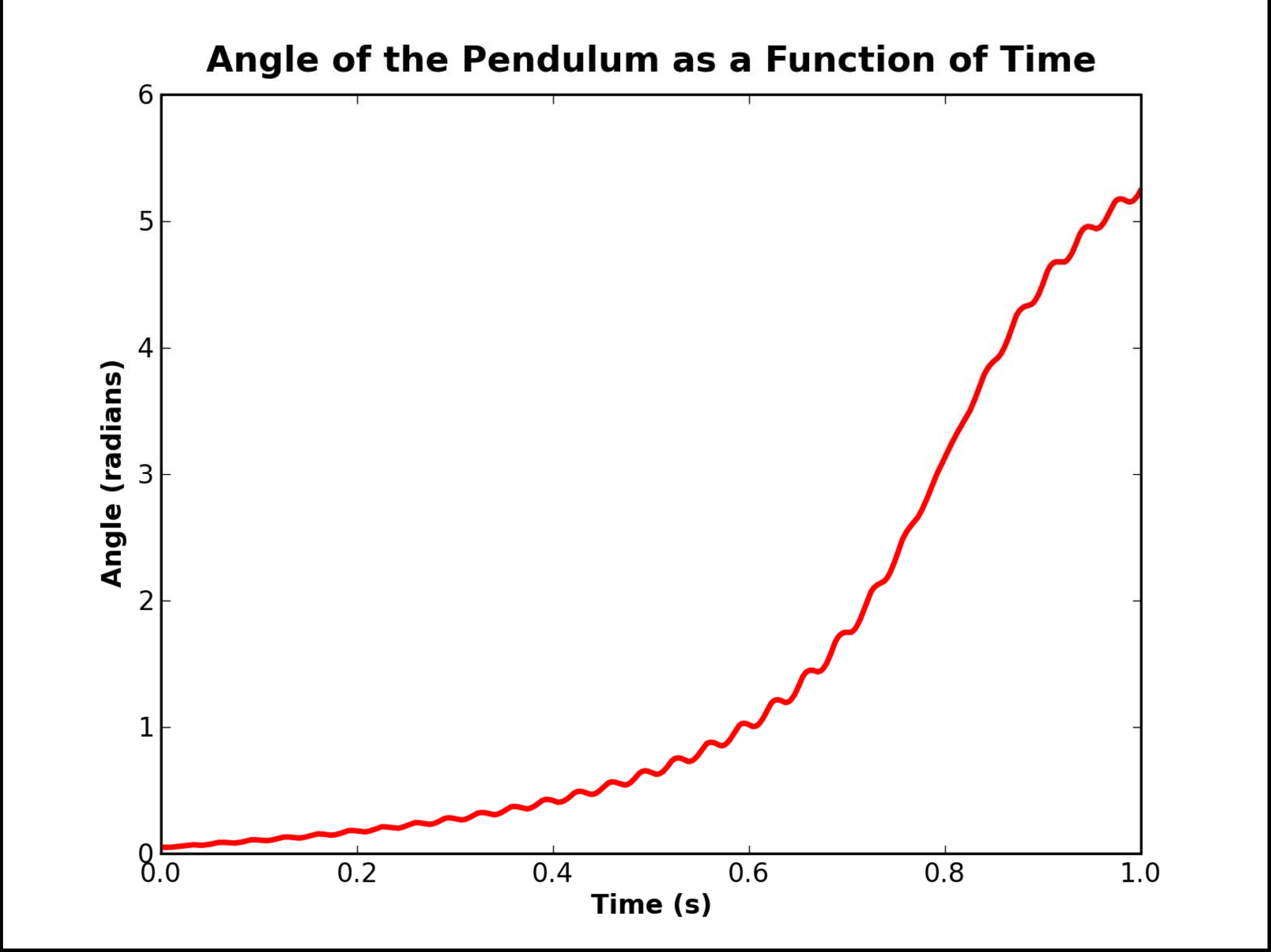
Results – 50Hz



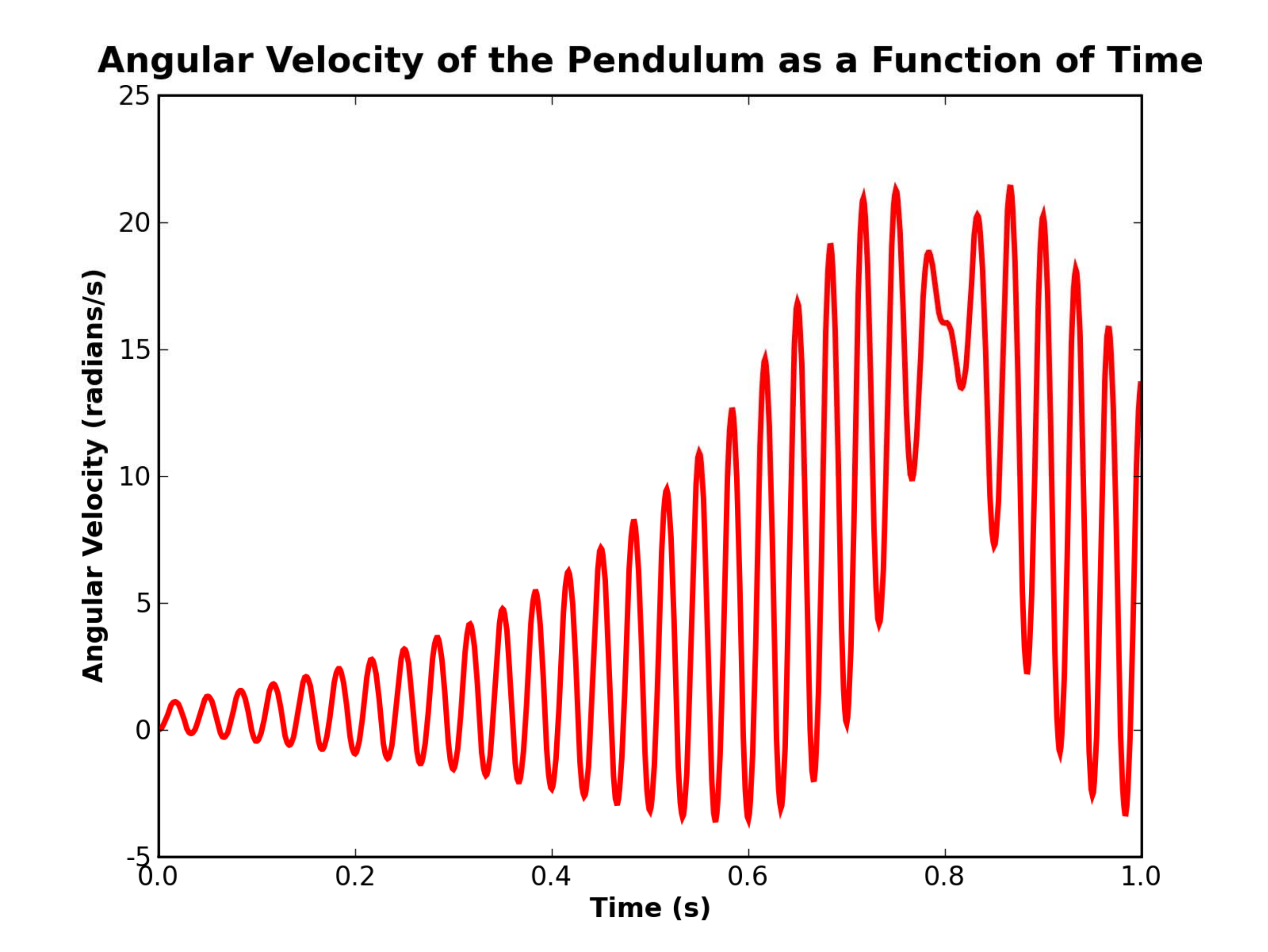
Results – 30Hz



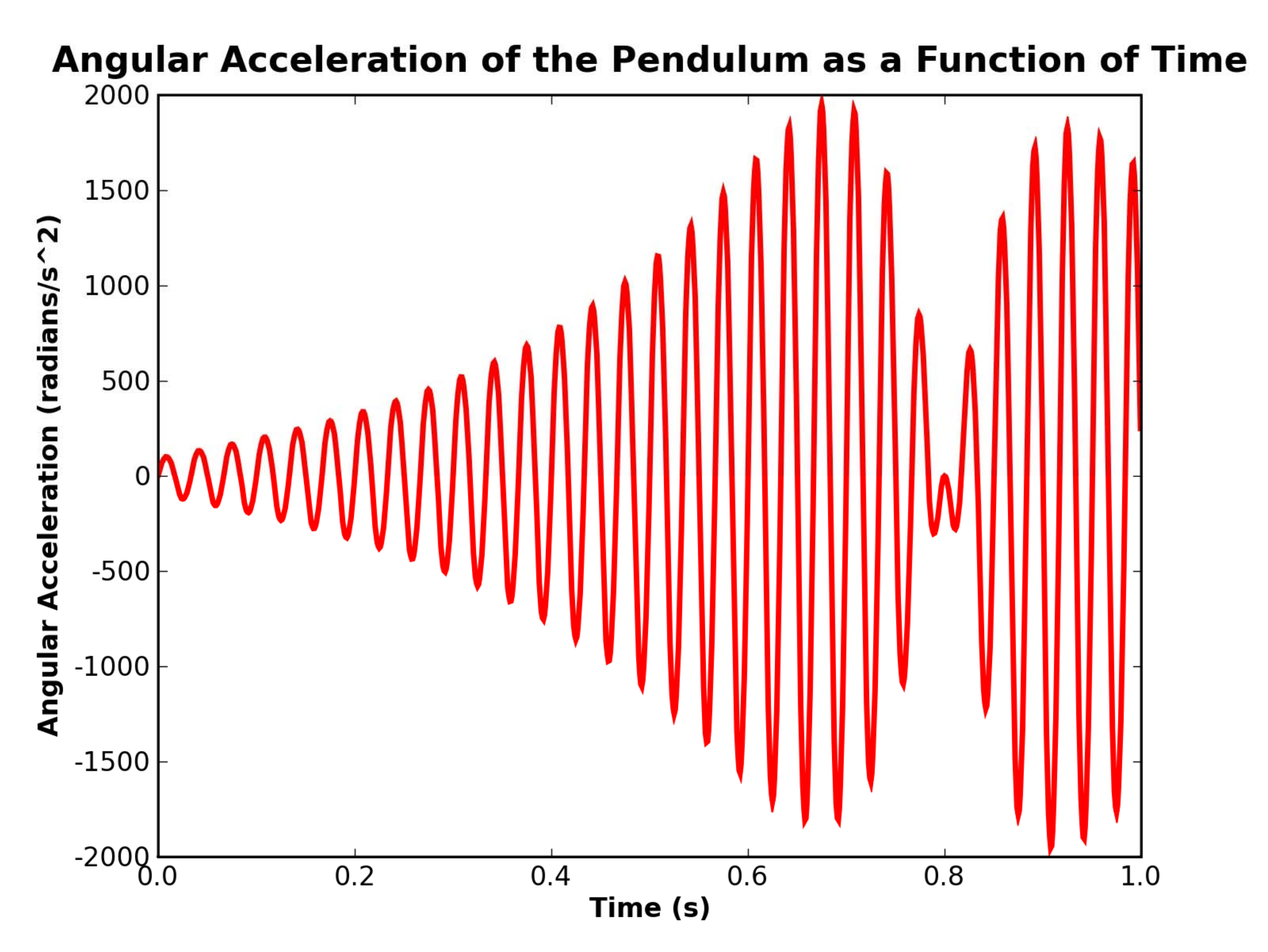
Results – 30Hz



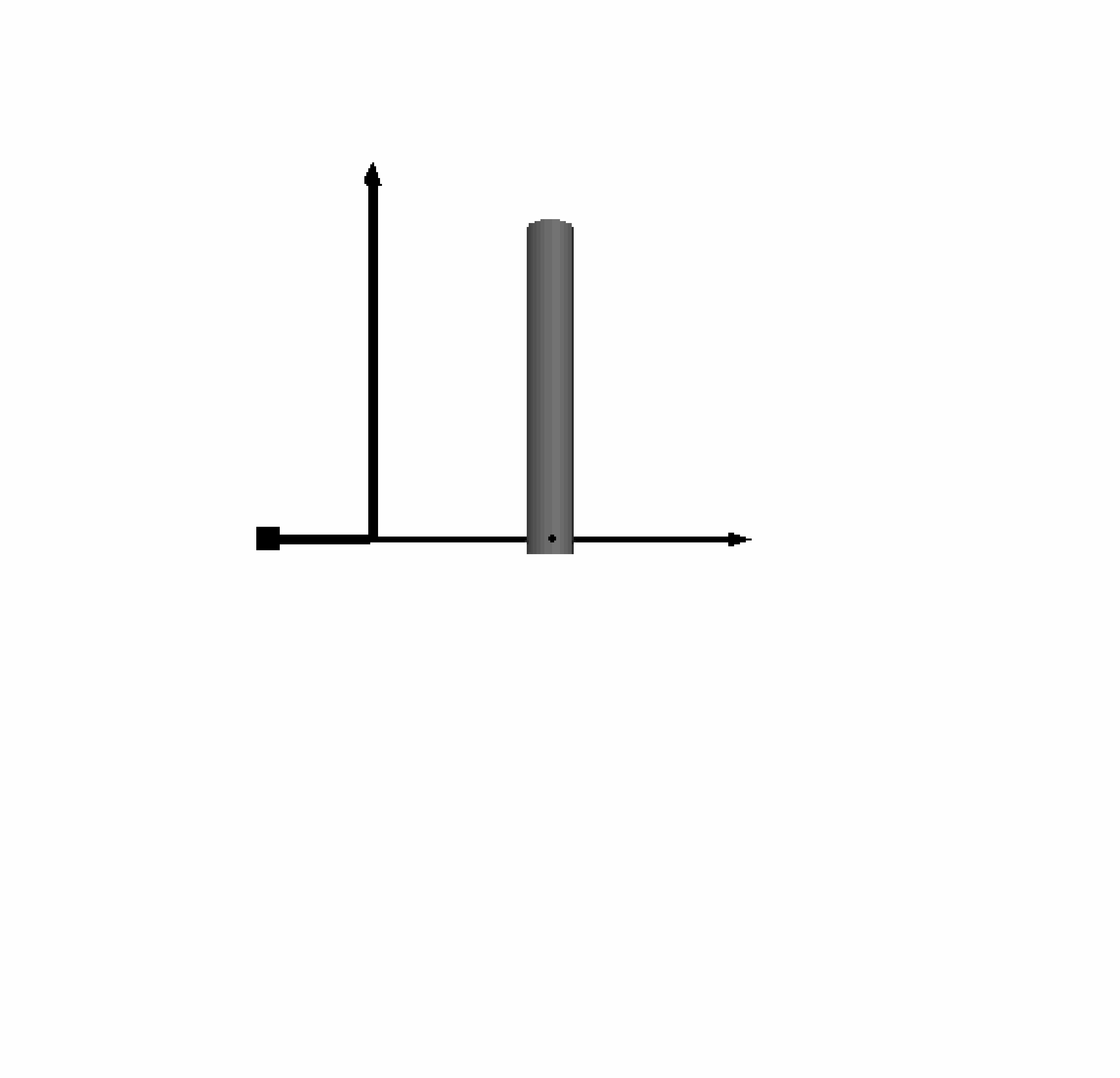
Results – 30Hz



Results – 30Hz



Results – 30Hz



Future Considerations

- Add the option to use square waves or triangular waves as the driving function, rather than only sinusoidal waves.
- Develop a better way to record simulated footage.
- Add complexity to the simulation.
  - Compound Pendulums

Conclusions

- Stability and frequency are very closely related.
  - If the frequency is too low or too high the pendulum will oscillate wildly.
- RK2 offers a blend of efficiency and simplicity.
- VPython is incredibly useful for simple animations.