Fourier Transforms in Optics

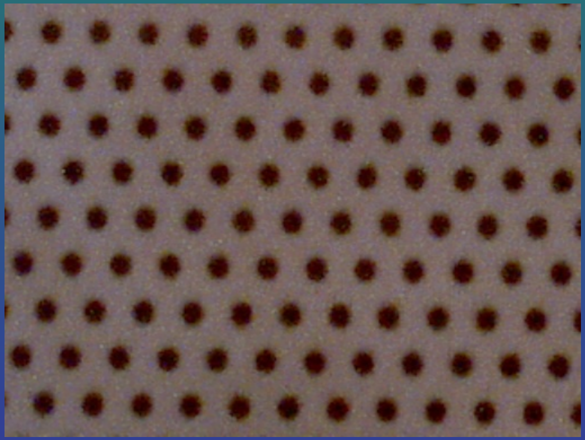
Definitions:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)e^{-ikx}dk \qquad F(k) = \int_{-\infty}^{\infty} f(x)e^{ikx}dx$$

- k is angular spatial frequency
- x is spatial variable (position)
- Fourier transforms are the inverse functions of one another
- They take you from real space to image space

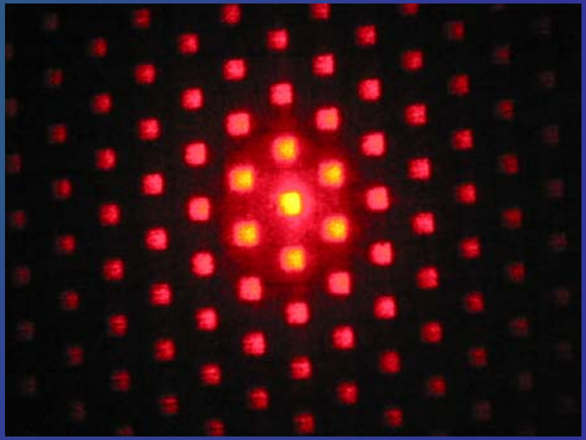
Joseph Fourier

http://en.wikipedia.org/wiki/Image:Fourier.jpg



Real Space

Image Space



In optics, if you model your aperture by a function, then the Fourier transform of that function will give you the E field, which you then square to get the intensity pattern.

Fourier Optics – The Single Slit Example

$$E(Y,Z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(y,z)\, e^{ik(Yy+Zz)/R}\, dy\, dz$$

For single slit:

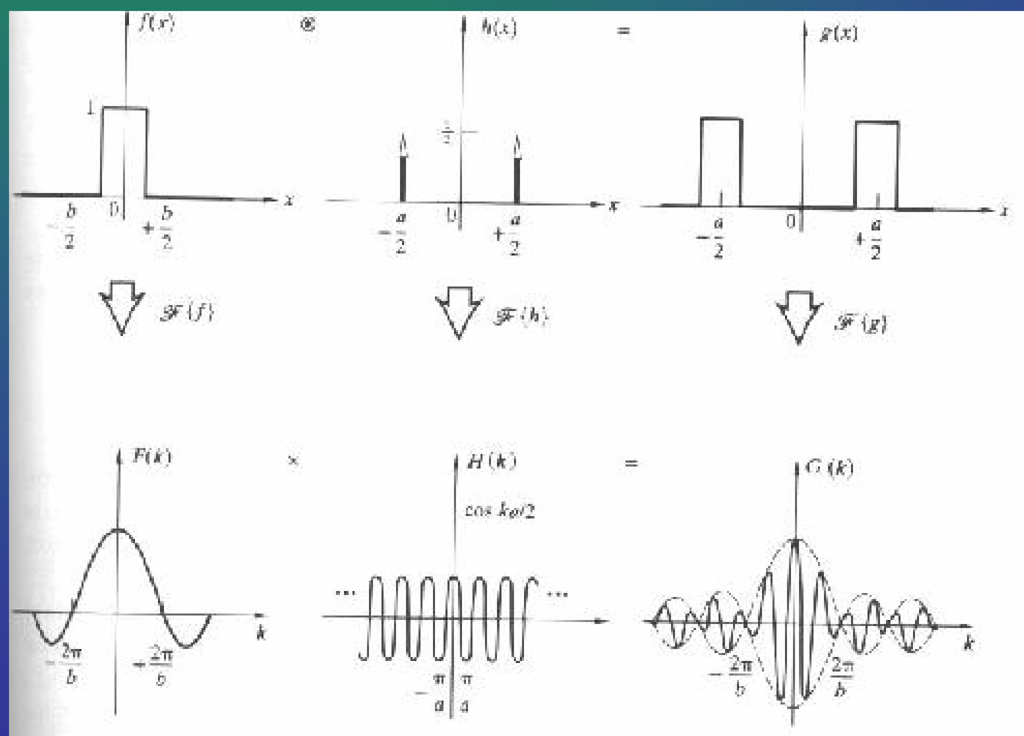
$$A(y,z) = \begin{cases} A_0 & \text{when } |z| \le b/2 \\ 0 & \text{when } |z| > b/2 \end{cases}$$

$$E(k_z,k_y) = \mathcal{F}\{A(y,z)\} = \int_{y=-b/2}^{+b/2}\int_{z=-a/2}^{+a/2} A_0 e^{i(k_y y + k_z z)}\, dy\, dz$$

$$E(k_y,k_z) = A_0 ba\, \mathrm{sinc}(bkY/2R)\, \mathrm{sinc}(akZ/2R)$$

Single Slit aperture function →

Single Slit E-field →



← Double Slit aperture function

← Double Slit E-field

Hecht, *Optics*, 1998. Addison Wesley Longman, Inc.

Diffraction – Results