Diffraction and Fourier Optics

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Fourier Transforms in Optics

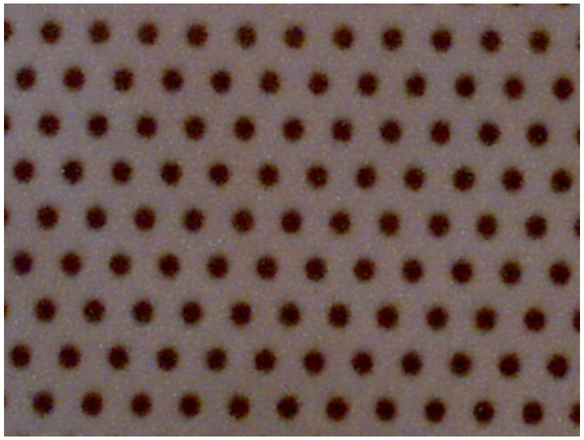
Definitions:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)e^{-ikx}dk \qquad F(k) = \int_{-\infty}^{\infty} f(x)e^{ikx}dx$$

- k is angular spatial frequency
- x is spatial variable (position)
- Fourier transforms are the inverse functions of one another
- They take you from real space to image space

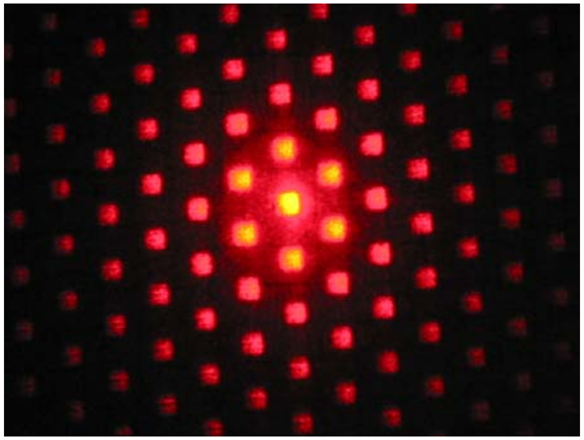
Joseph Fourier

http://en.wikipedia.org/wiki/Image:Fourier.jpg



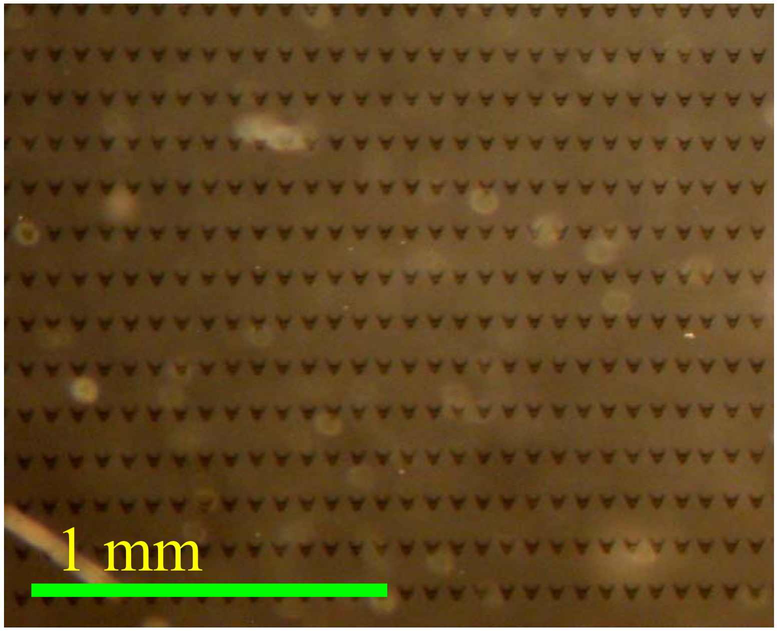
Real Space

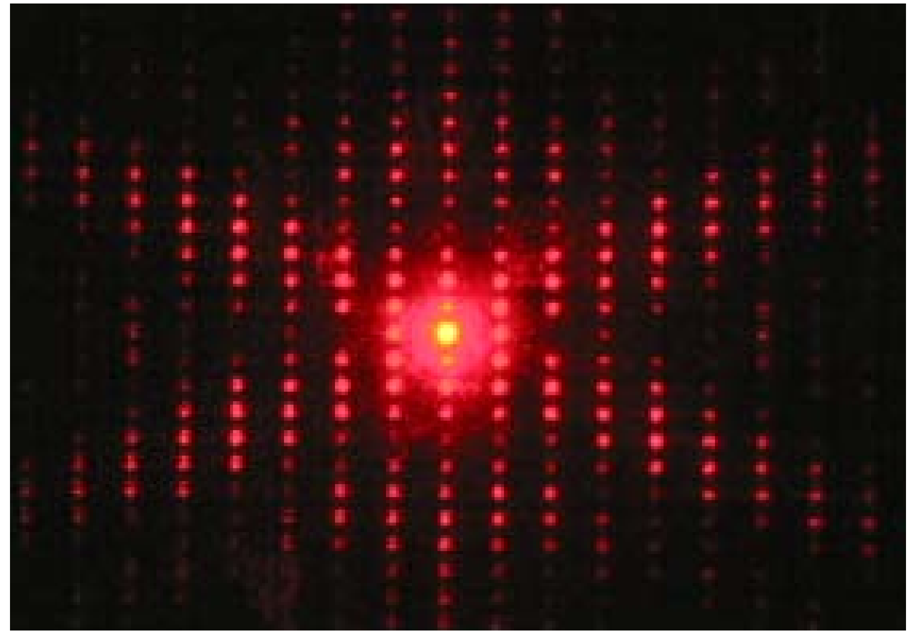
Image Space

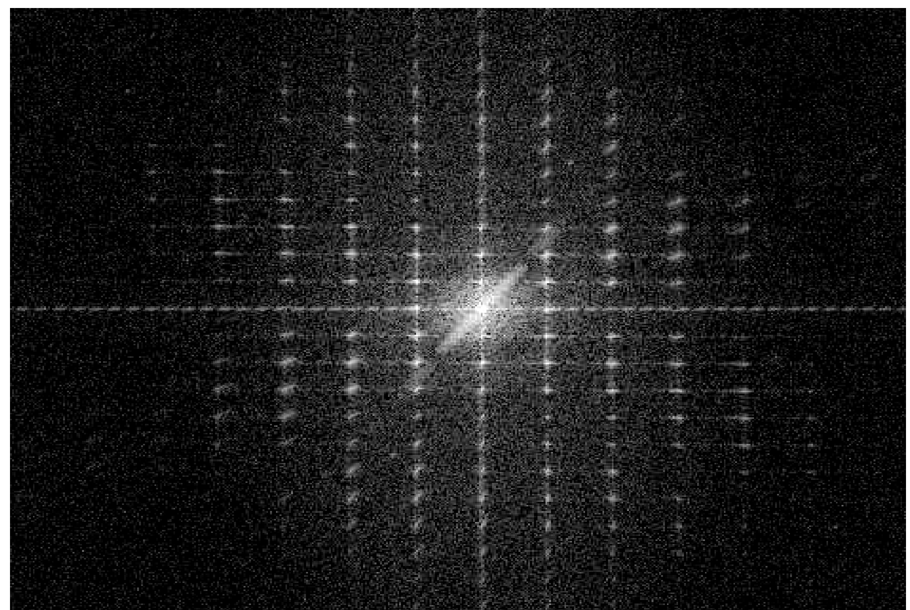


In optics, if you model your aperture by a function, then the Fourier transform of that function will give you the E field, which you then square to get the intensity pattern.

Real Space vs. Fourier Space







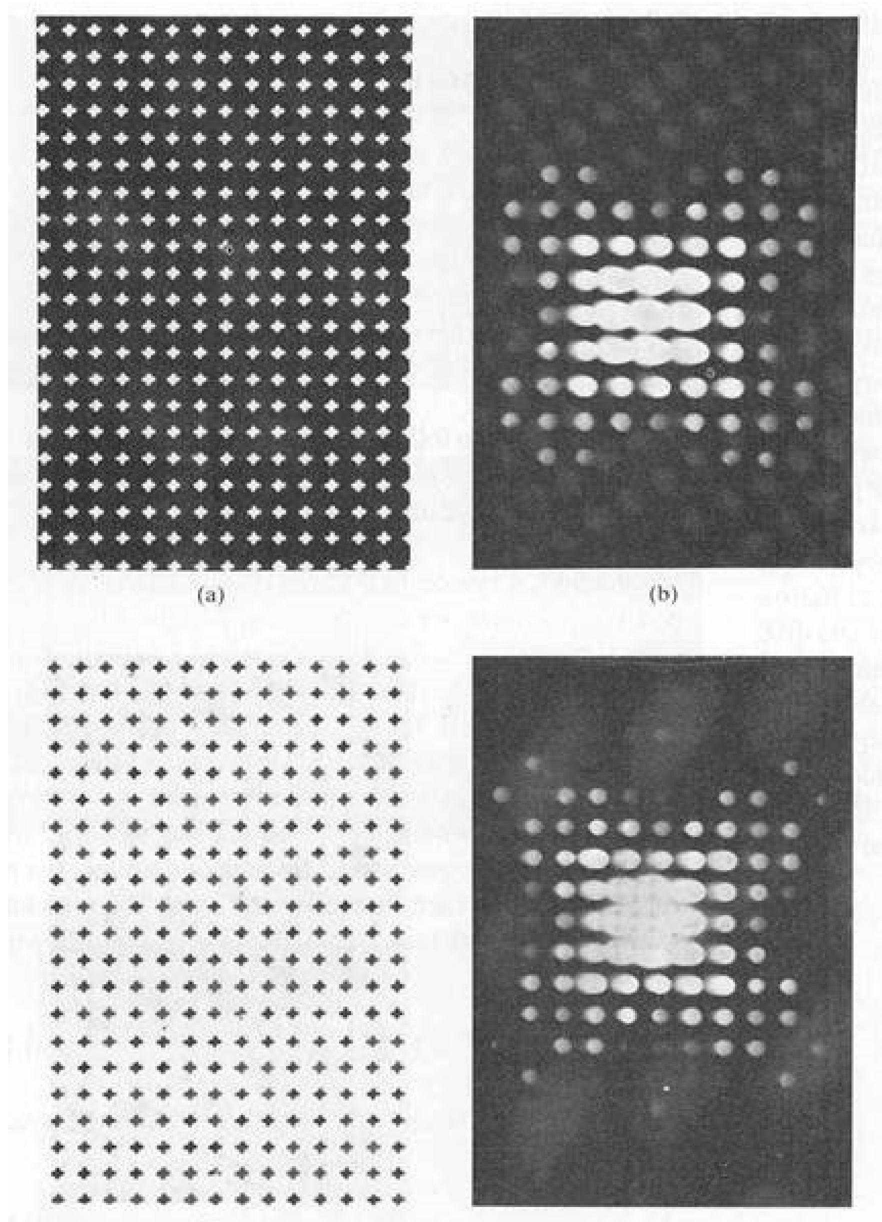
- (Top Left) Actual Slide
- (Top Right) Actual image in Fourier Space by shining laser through slide
- (Right) Predicted Fourier transform created in ImageJ

Babinet's Criteria

- Complementary gratings create the same interference pattern.
- Addition of the electric field intensity

$$E_1 + E_2 = E_{total}$$

- For these gratings E_{total} would be zero
- Most of the slides are printed onto, not cutouts



Fourier Optics – The Single Slit

$$E(Y,Z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(y,z)\, e^{ik(Yy+Zz)/R}\, dy\, dz$$

For single slit:

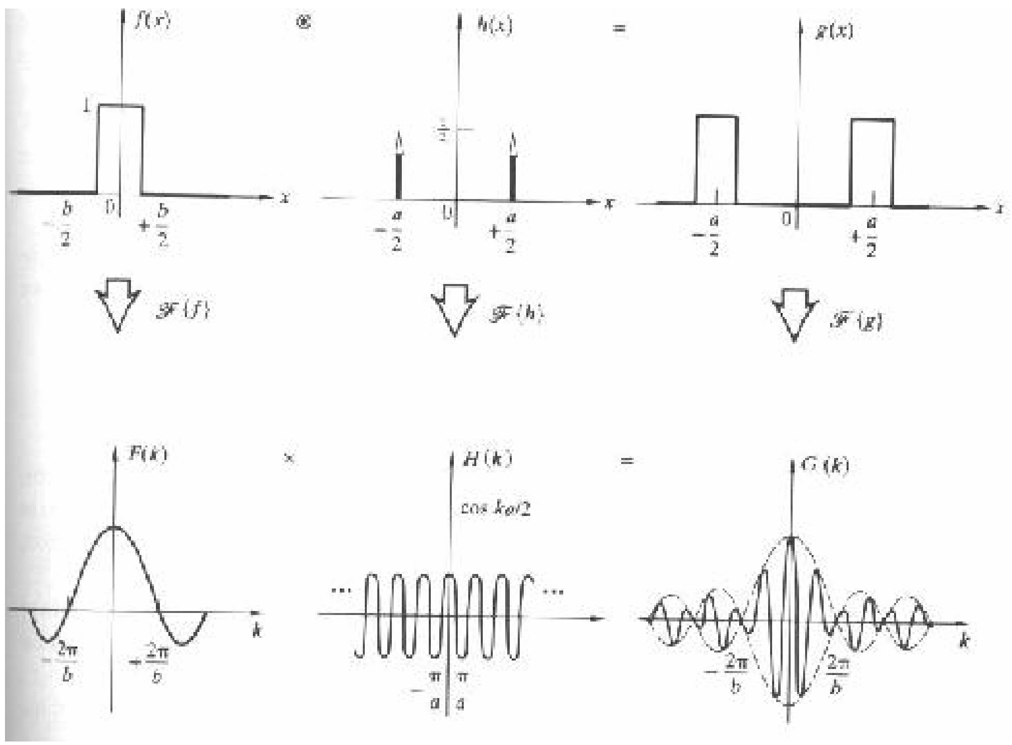
$$A(y,z) = \begin{cases} A_0 & \text{when } |z| \le b/2 \\ 0 & \text{when } |z| > b/2 \end{cases} \longrightarrow$$

$$E(k_z,k_y) = \mathcal{F}\{A(y,z)\} = \int_{y=-b/2}^{+b/2}\int_{z=-a/2}^{+a/2} A_0 e^{i(k_y y + k_z z)}\, dy\, dz$$

$$E(k_y,k_z) = A_0 ba \,\text{sinc}(bkY/2R)\,\text{sinc}(akZ/2R)$$

Single Slit aperture function →

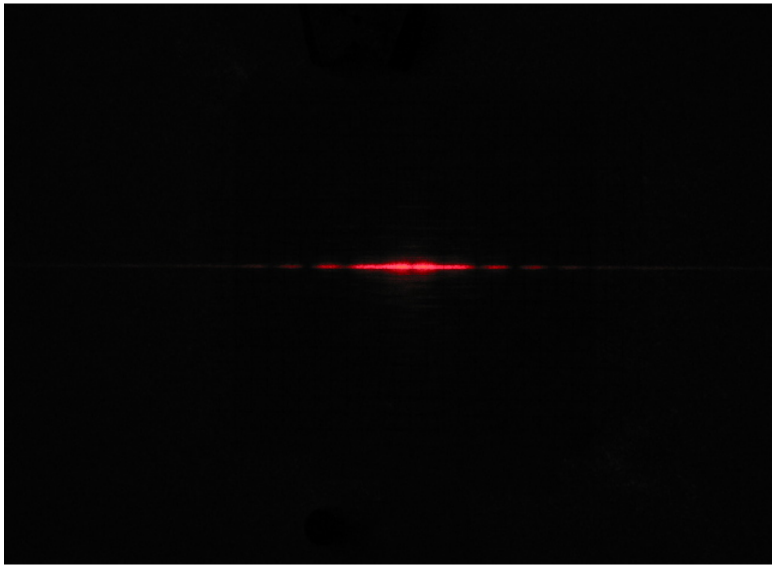
Single Slit E-field →



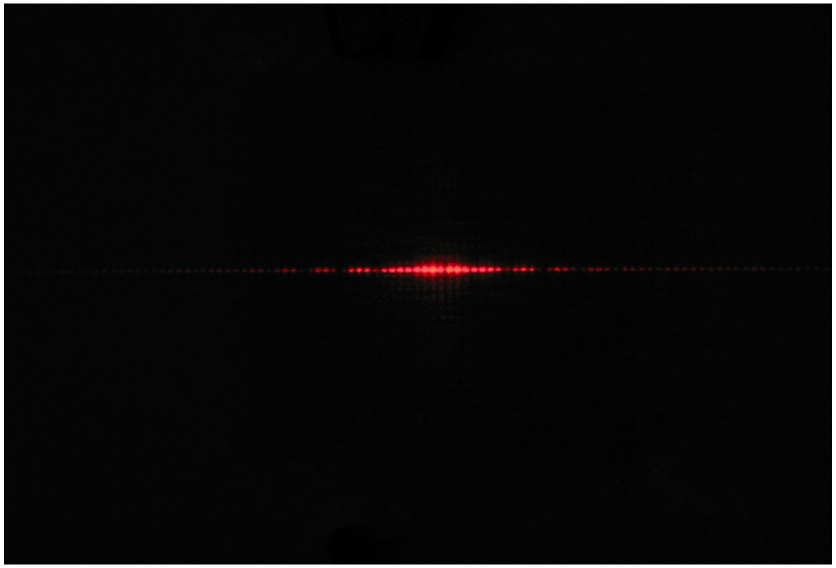
← Double Slit aperture function

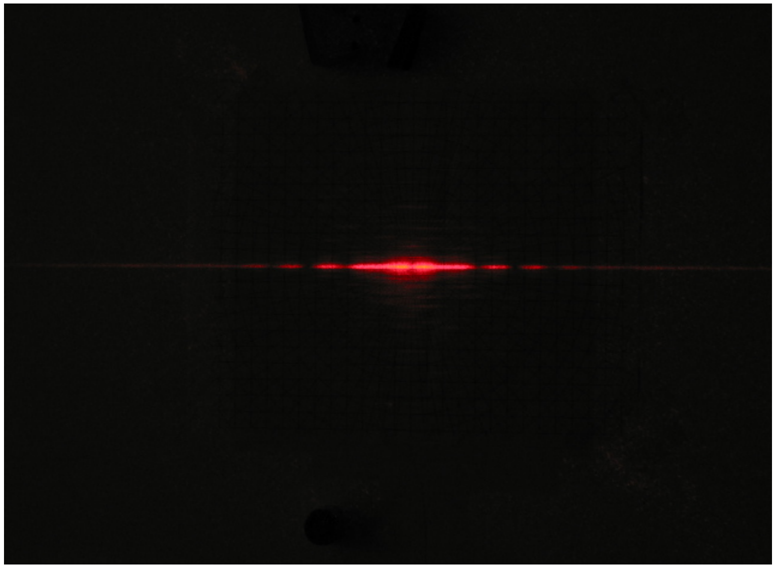
← Double Slit E-field

Single Slit | Double Slit

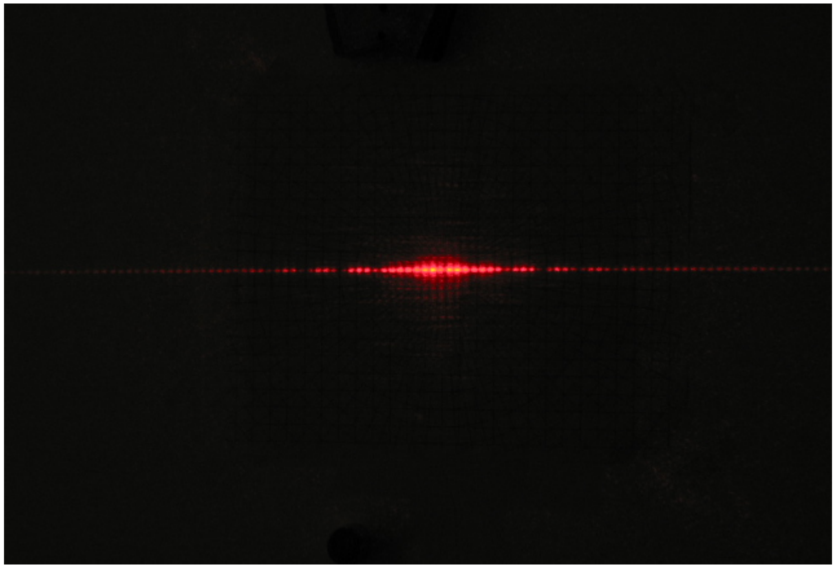


4s



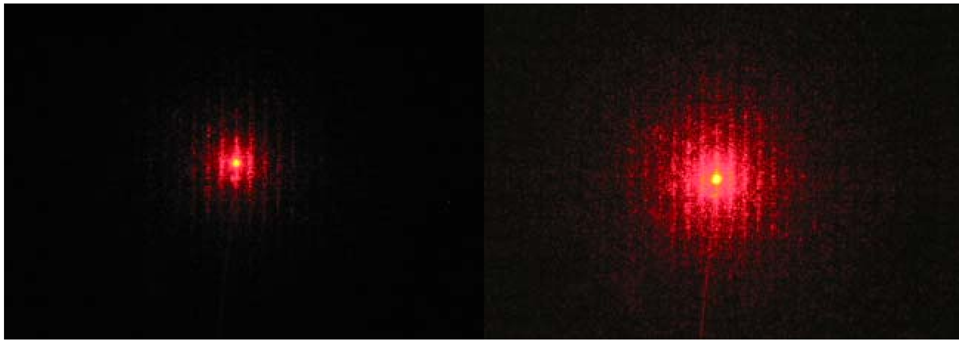


8s

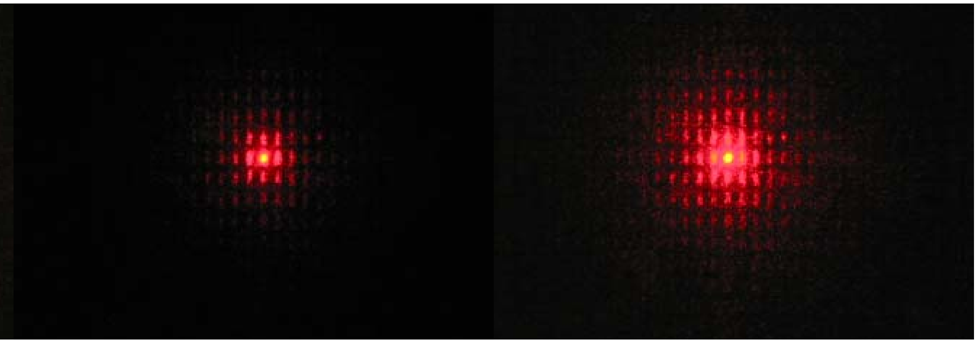


- Two slits leads to an additional diffraction pattern within the original diffraction envelope

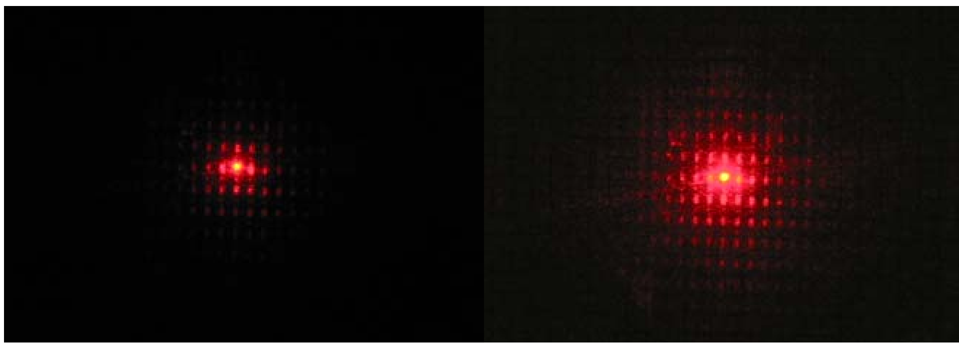
One Row – Dots



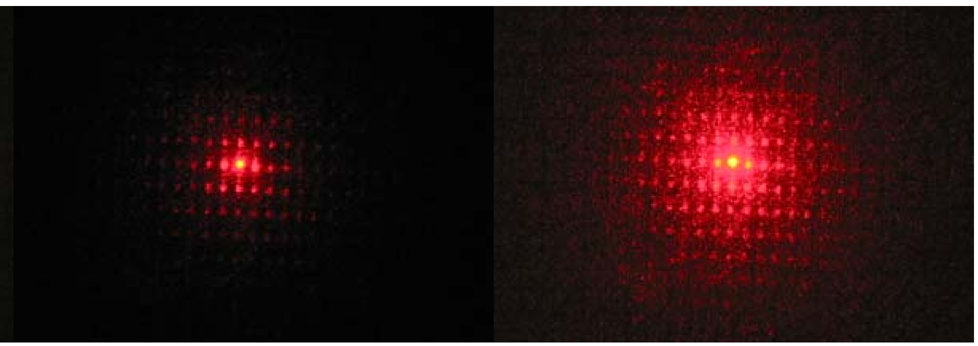
Two Rows - Dots



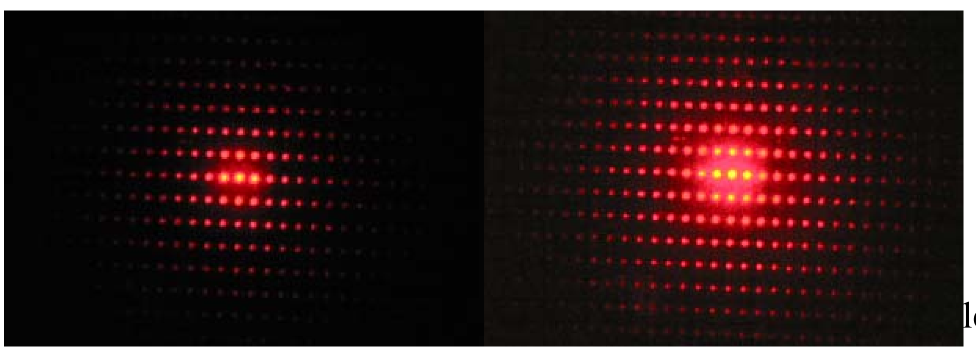
Three Rows – Dots



Four Rows – Dots

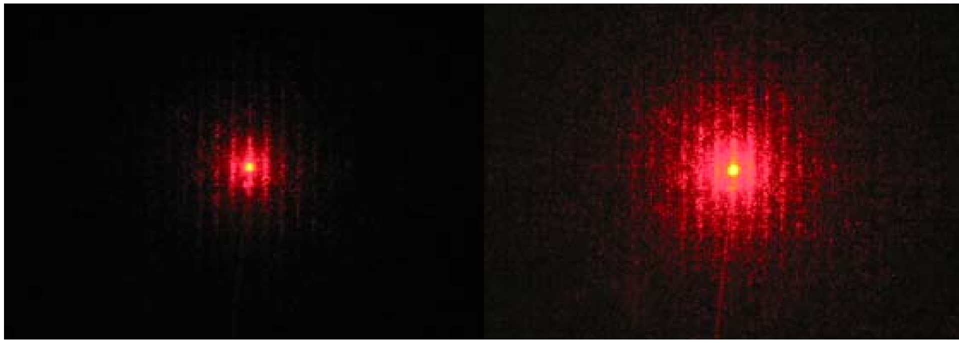


Rectangular Array – Dots



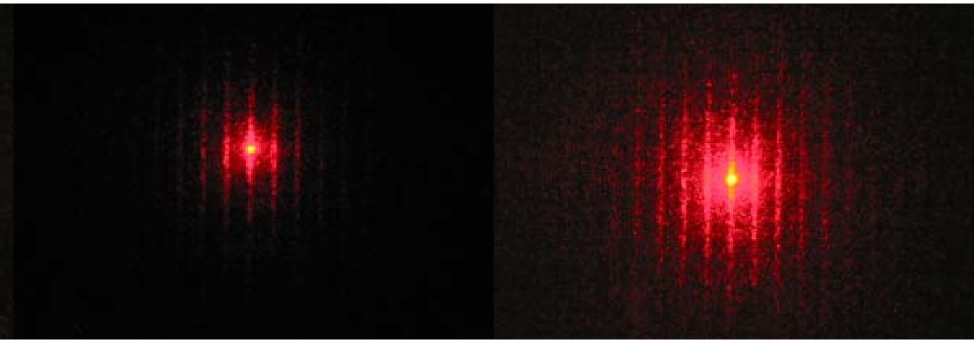
- As more rows are horizontally stacked, the image localizes more and more vertically
- All rows have 0.12mm spacing
- The first images were taken with 4s exposure time, and the second images were exposed for 6s

One Row – Dots – 0.12mm



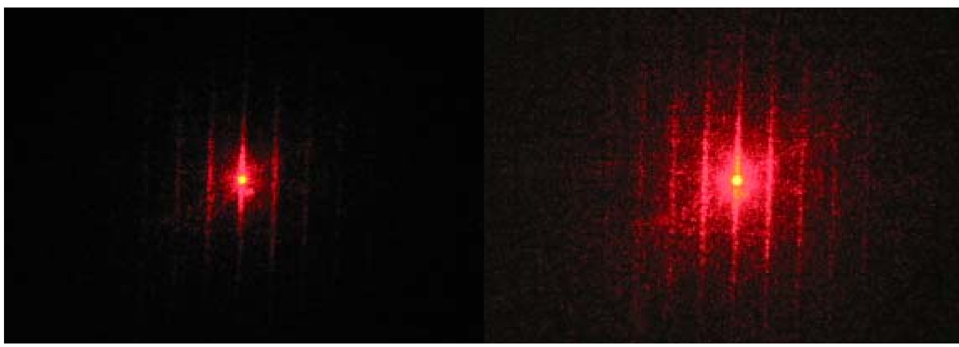
4s 6s

One Row – Dots – 0.08mm



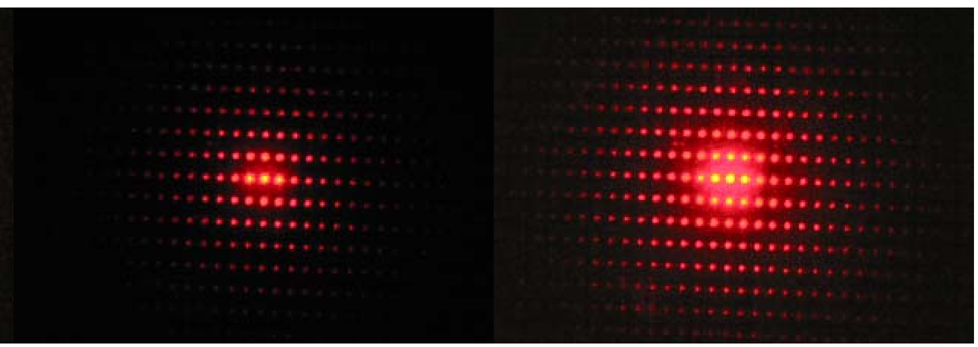
4s 6s

One Row – Dots – 0.06mm



4s 6s

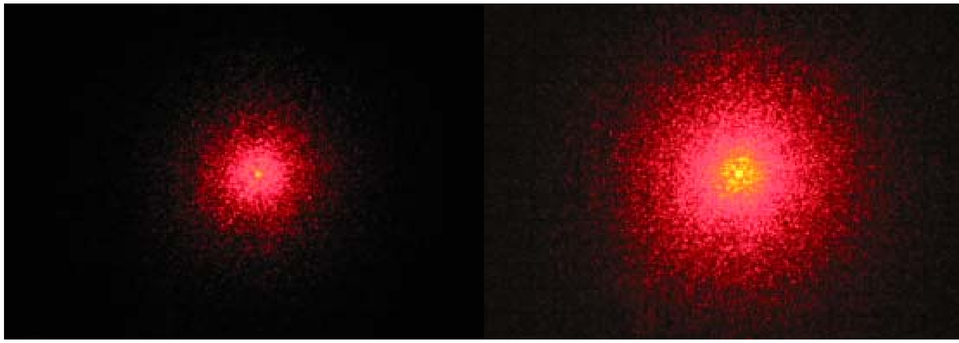
Rectangular Array – Dots - 0.12x0.08mm



4s 6s

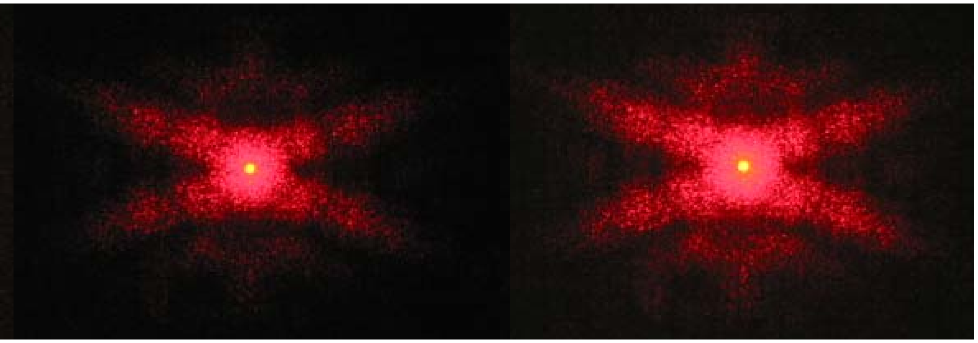
- Wider spacing in real space = narrower spacing in image Fourier space

Random Dots



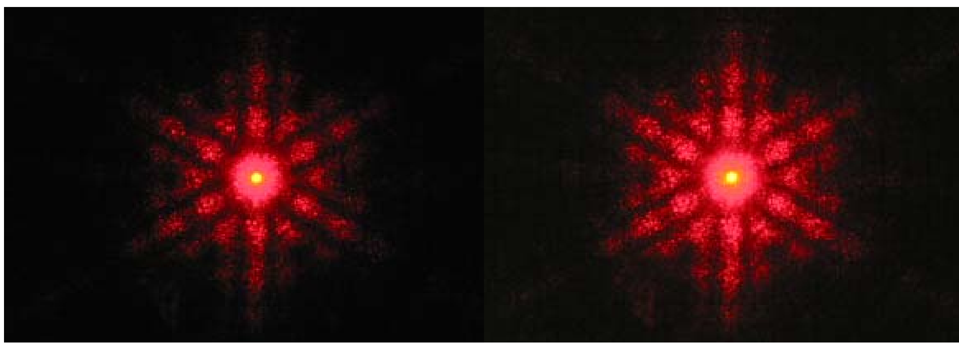
1s 6s

Random A's



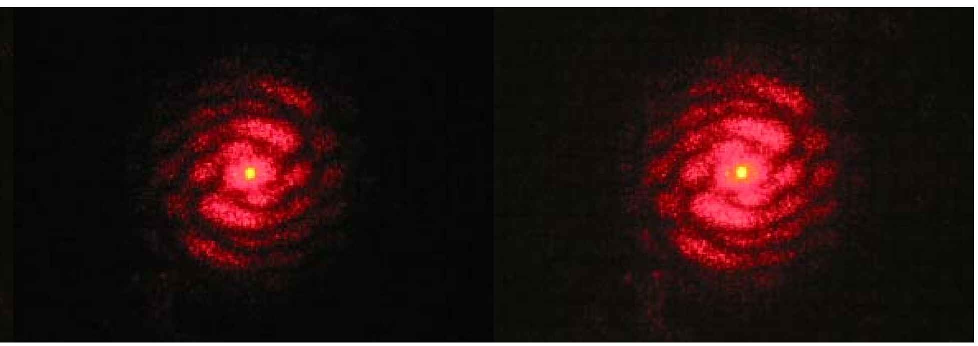
4s 8s

Random *'s



4s 8s

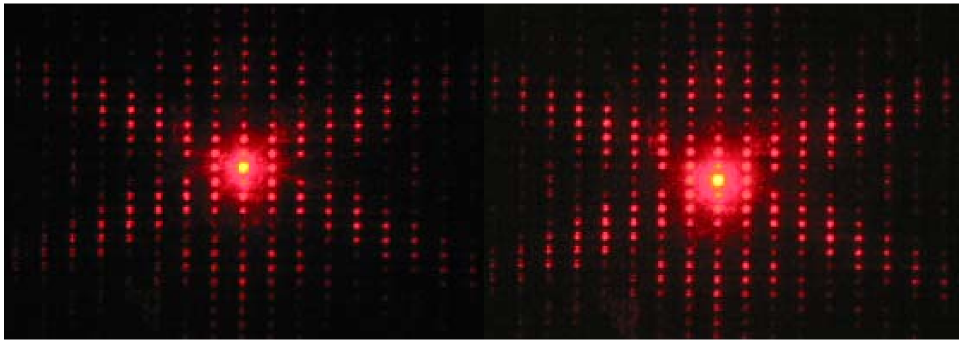
Random S's



4s 8s

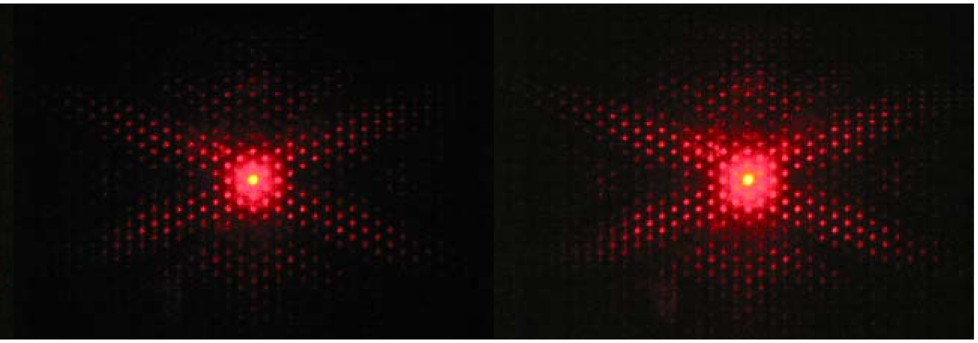
- The large-scale diffraction pattern is the Fourier transform of the kind of object the light diffracts off of
- Symmetry is clearly derived from the shape of the object

Rectangular A’s



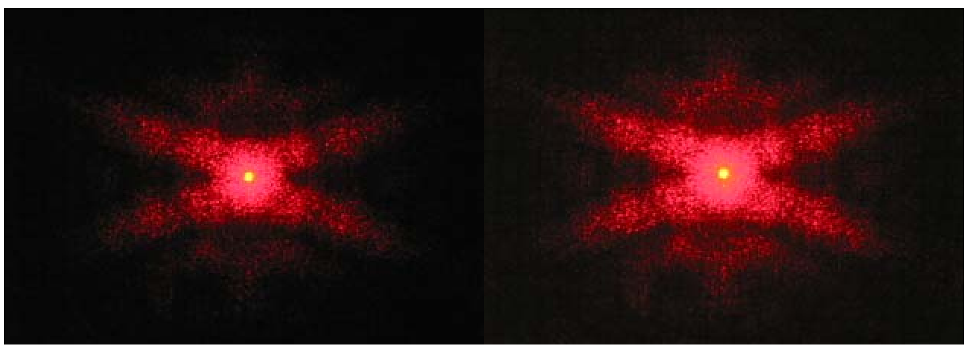
4s 8s

Hexagonal A’s



4s 8s

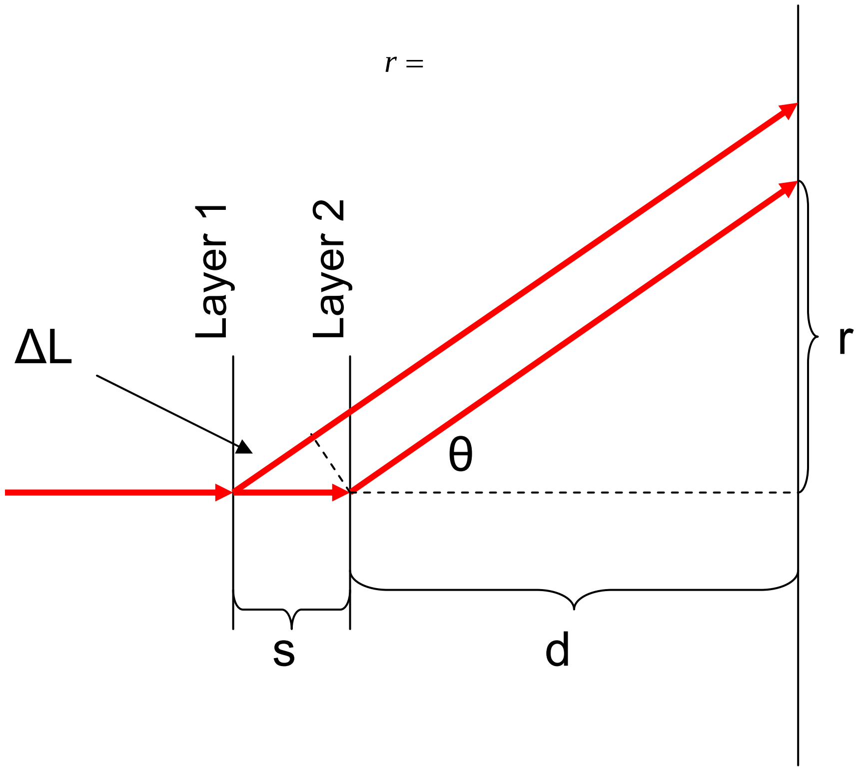
Random A’s



4s 8s

- The kind of symmetry of the placement of the A’s leads to patterns within the large-scale diffraction pattern
- Symmetry in real space shows up with the same kind of symmetry in Fourier space

Double Layer Diffraction



$$s << d$$

- ΔL is the difference in path length between light from Layer 1 and Layer 2
- Fringes observed reach a minimum when the light is ½ a wavelength out of phase
- s - ΔL = (n+1/2)λ, n is a positive integer
- Cosθ = ΔL/s = 1 - (n+1/2)λ/s at minimum fringe points
- The radial distance from center of screen is r = dtanθ

Minimum fringes should occur at $r = d\tan(\cos^{-1}(1 - \frac{(n+\frac{1}{2})\lambda}{s}))$

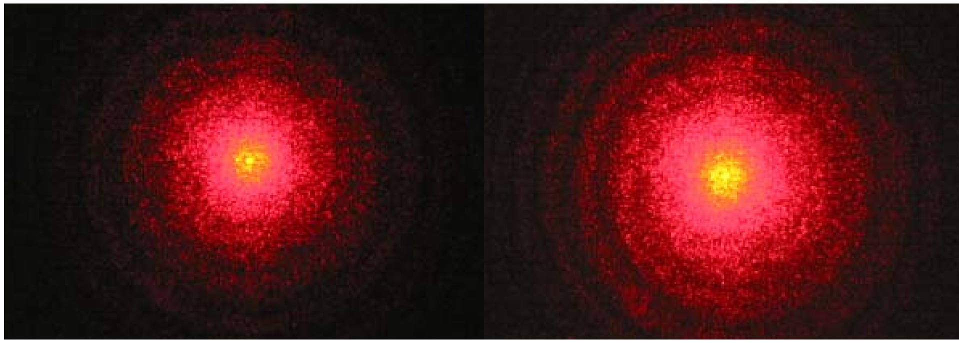
Double Layer Diffraction - Model

	Rectangular Array (.08 x .12 mm)	Rectangular Array (.12 x .18 mm)	Random Array	Hexagonal Array (.1 mm)	Average
1	4.90	5.00	5.30	6.30	5.375
2	7.20	7.00	7.00	7.88	7.27
3	8.70	8.50	8.70	9.50	8.85
4	10.40	10.00	10.20		10.20
5	11.70	11.40	11.20		11.43

Constants	in	cm			mm
R	54.00	137.16		**L**	0.088
				λ	6.33E-05

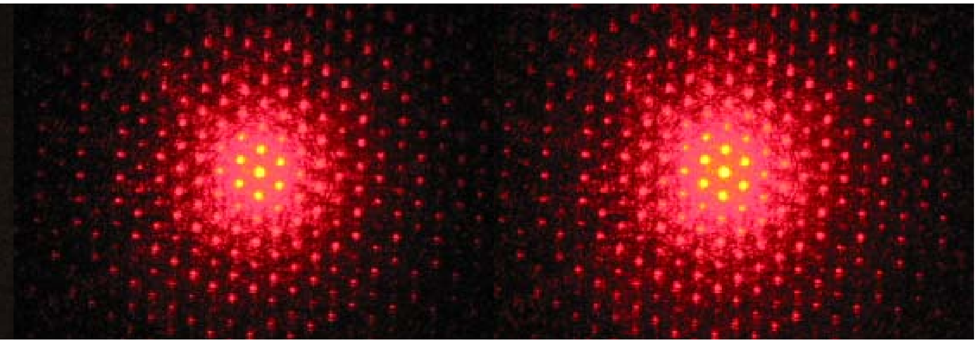
Model				
n	**y**		**Measurements**	**Percent Error**
	1	6.38E+00	5.38	1.57E-01
	2	8.24E+00	7.27	1.17E-01
	3	9.75E+00	8.85	9.24E-02
	4	1.11E+01	10.20	7.80E-02
	5	1.22E+01	11.43	6.57E-02

Two Layer Random Dot's



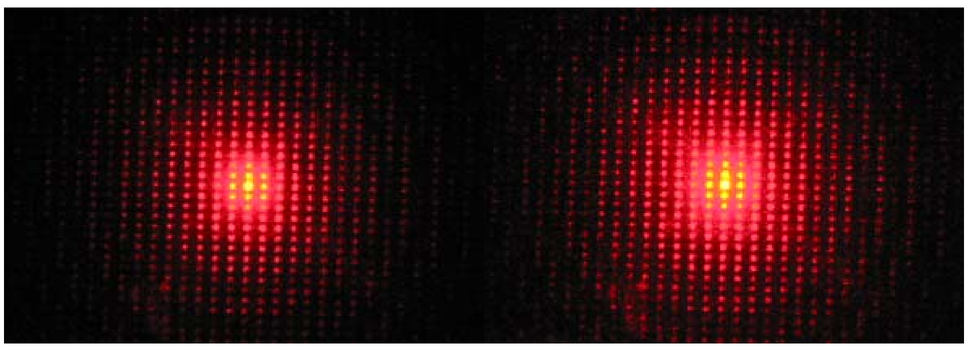
6s 13s

Two Layer – Hexagonal



8s 13s

Two Layer – Rectangular



8s 15s

- The double-plane causes circular diffraction due to the phase difference between the planes