Chladni Plates

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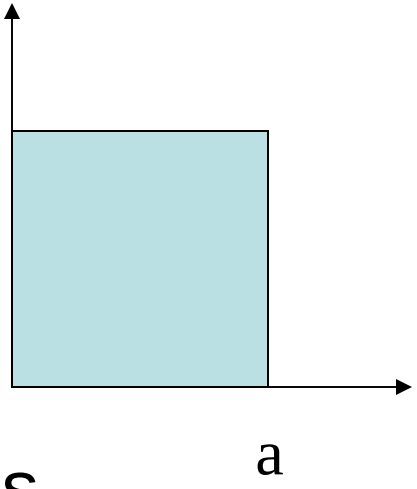
History

- Ernest Chladni: Father of acoustics
- Vibrated fixed circular plates with violin bow
  - Sprinkled sand on the plates
  - Sand settles in the nodes on the plate
  - Can observe nodal lines and patterns
- Complex patterns result for very high frequencies
- Chladni's Law:
  - $f \sim (m + 2n)^2$

Square Plate Theory

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}$$



- Governed by 2D wave equation
- Analogous to 1-D pipe open at both ends
- Separation of variables
  - Look for solution form:
- Solutions: $U(x,y,t) = X(x)Y(y)T(t)$
  - X: $X(x) = A\sin(k_x x) + B\cos(k_x x)$
  - Y: $Y(y) = C\sin(k_y y) + D\cos(k_y y)$
  - T: $T(t) = e^{i\omega t}$
- Boundary Condition:
  - Must be anti-node at plate edge

Square Plate Theory Continued

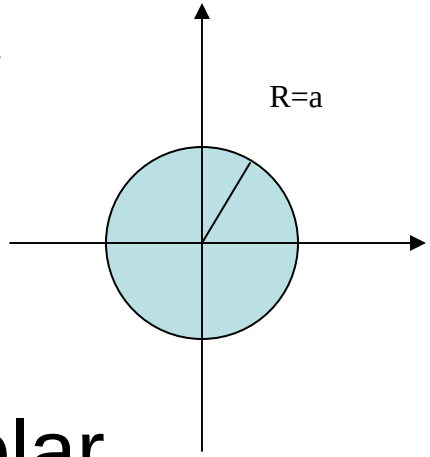
- Applying boundary conditions
  - A and B=0

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{a}$$

- Solution: $U(x,y,t) = D\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{a}\right)e^{i\omega t}$
  - Where: $\omega = vk$ and $k^2 = k_x^2 + k_y^2$

- Frequency: $\omega = \frac{v\pi}{a}\left(m^2 + n^2\right)^{\frac{1}{2}}$

Circular Plate Theory

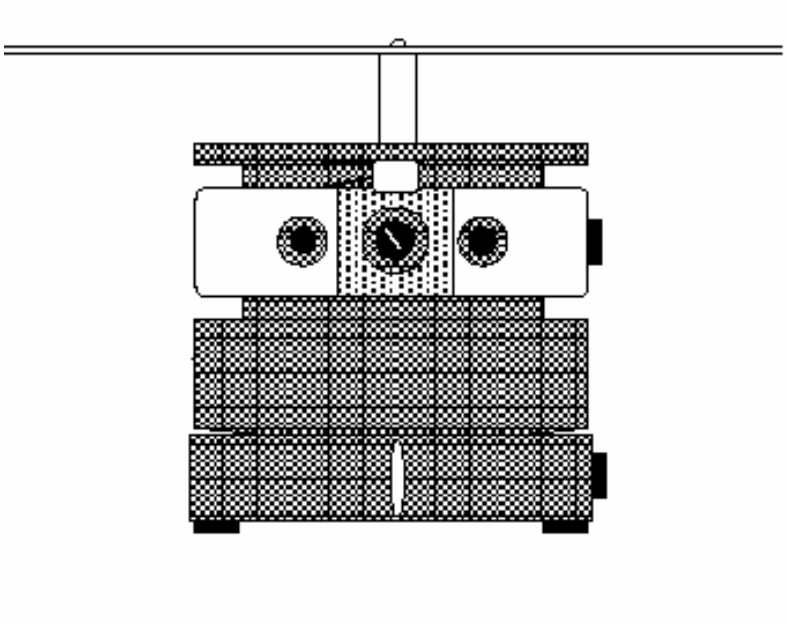


$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial \phi^2}=\frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}$$

- Governed by wave equation in polar coordinates
- B.C.: Anti-node at edge
- Separation of variables and look for solution of form: $U = R(r)\Phi(\phi)T(t)$
  - Eq.1: $\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\left(k^2-\frac{m^2}{r^2}\right)R = 0.$
  - Eq.2: $\frac{d^2\Phi}{d\phi^2}+m^2\Phi = 0.$
  - Eq3: $\frac{d^2T}{dt^2}+k^2c^2T = 0.$
- Solution: $U(r,\phi,t) = J_m(k_n r)\cos(m\phi)e^{i\omega_n t}$

The Experiment

- Equipment
  - PASCO Chladni Plates Kit



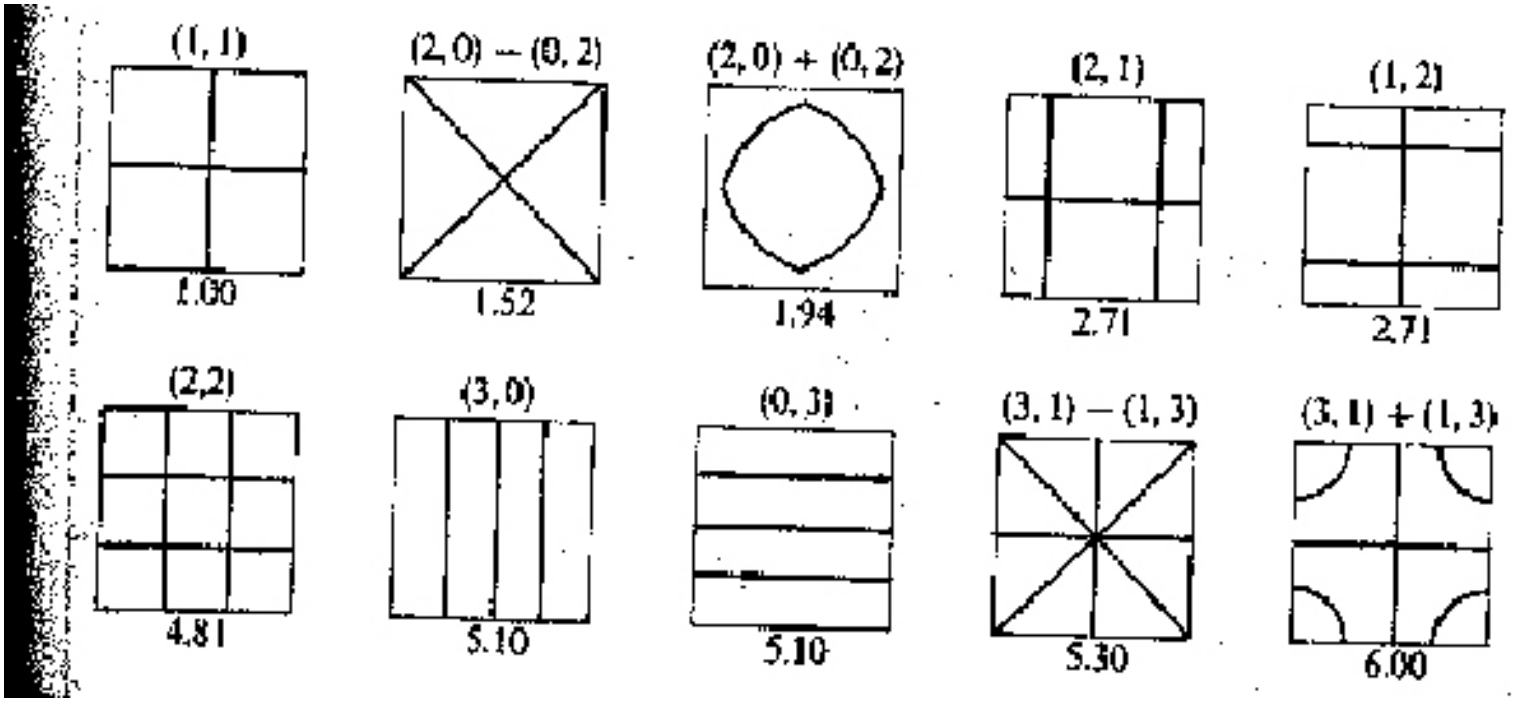
Mechanical Wave Driver

Procedure

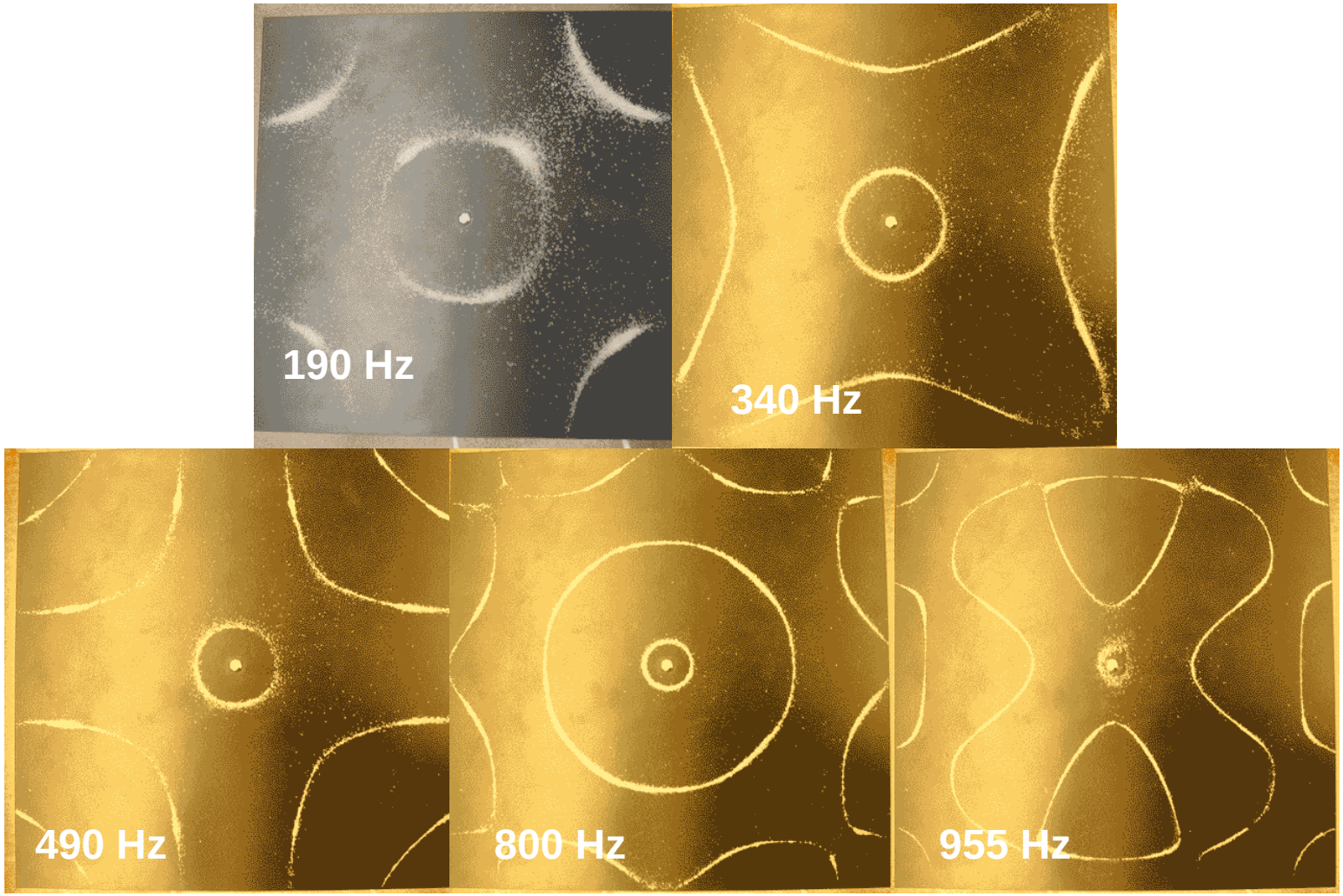
- The mechanical driver is attached to the plates at the center
  - We use both a circular and a square plate
  - The circular plate was also driven off-center
- By adjusting the frequency of the generator we can observe resonances
- Sand collects at the nodes and forms patterns
- We weren’t able to observe all possible patterns
- Also tried materials of different consistency (baby powder, baking soda)
  - In theory this should produce different patterns. Why??

Square Plate

- m and n values were not discernible
- Also, the patterns we observed were very different from the predicted patterns..
- The predicted patterns are given in The Physics of Musical Instruments by Fletcher and Rossing

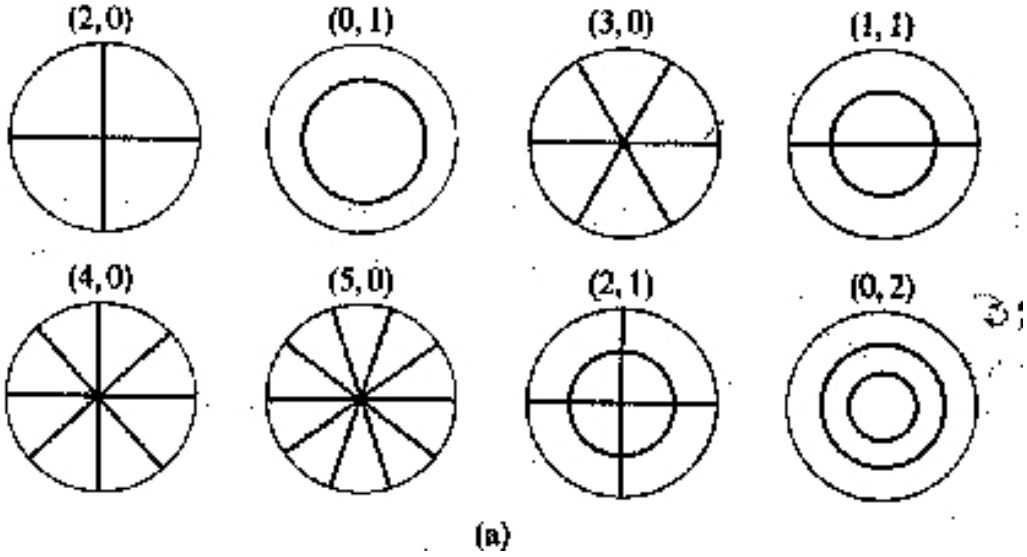


Square Patterns Observed..

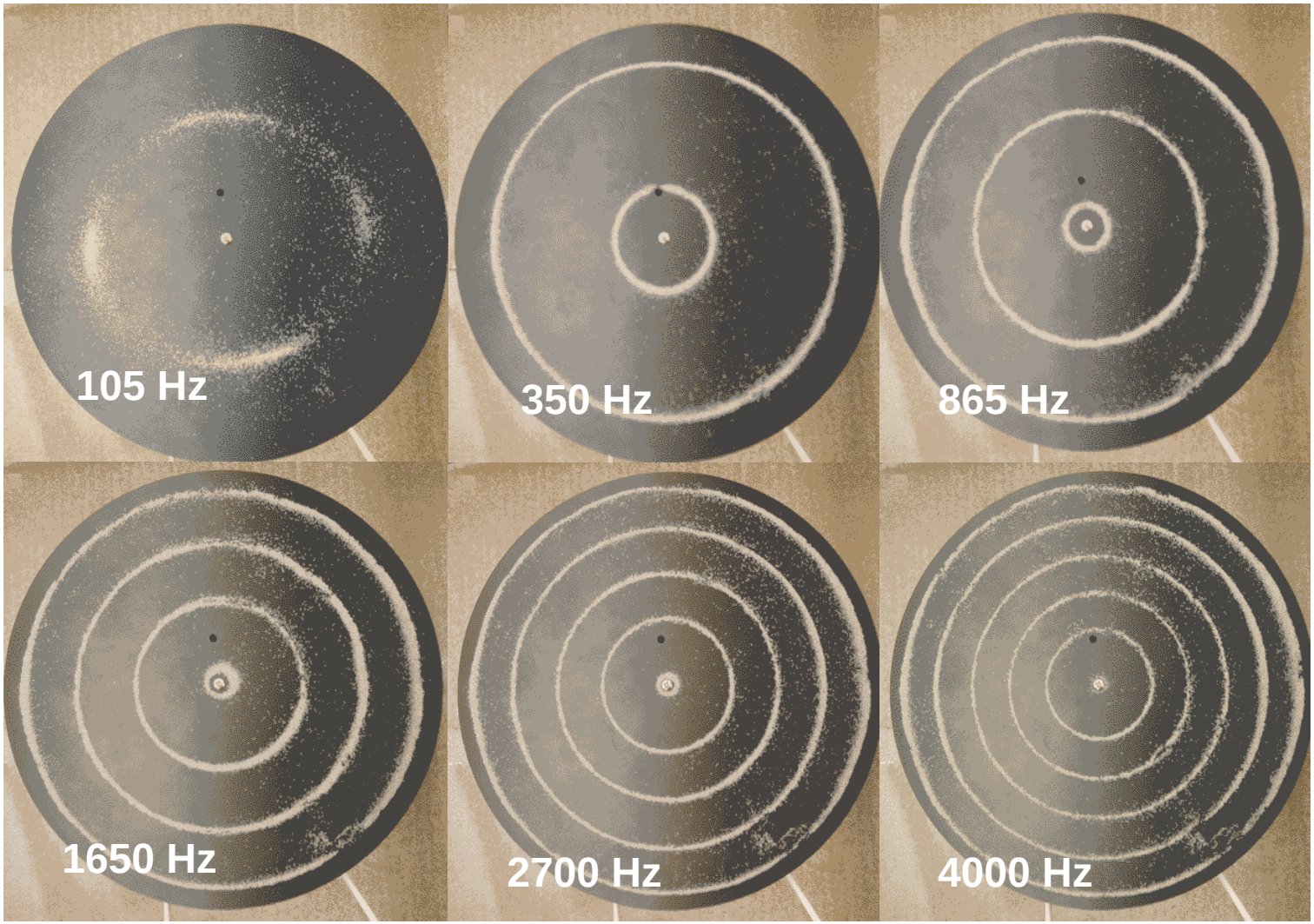


Circular Plates

- n values were easy to obtain (n=1 for 1 concentric circle, etc.)
- m values (nodal diameters) did not show up often
  - Probably because the plate is being driven at the center
  - Easier to see for plates with fixed edges, as well
- Each resonance we observed added another concentric circle..not always a node at center
- Predicted patterns are given, again, by Fletcher and Rossing

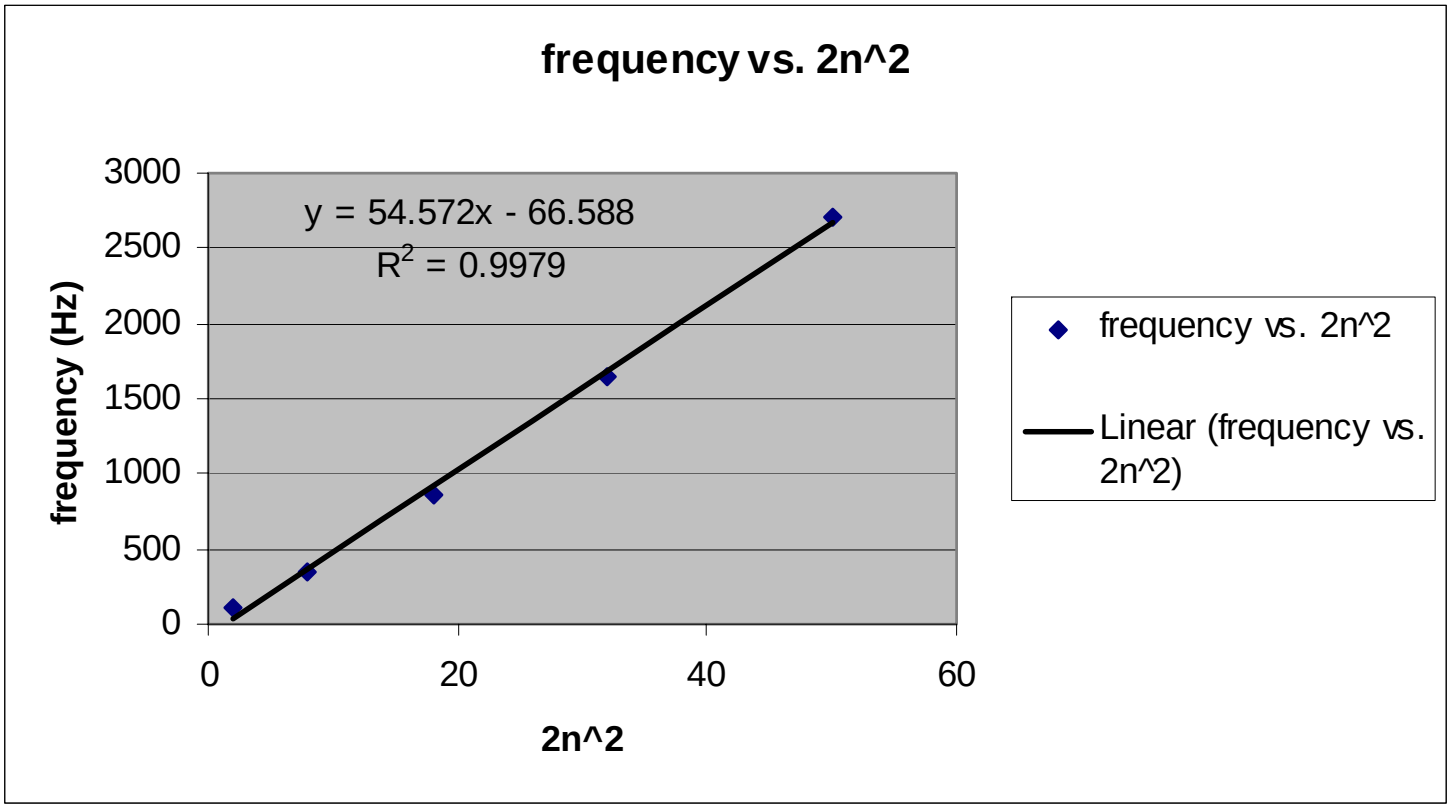


Circular Patterns Observed



Chladni's Law

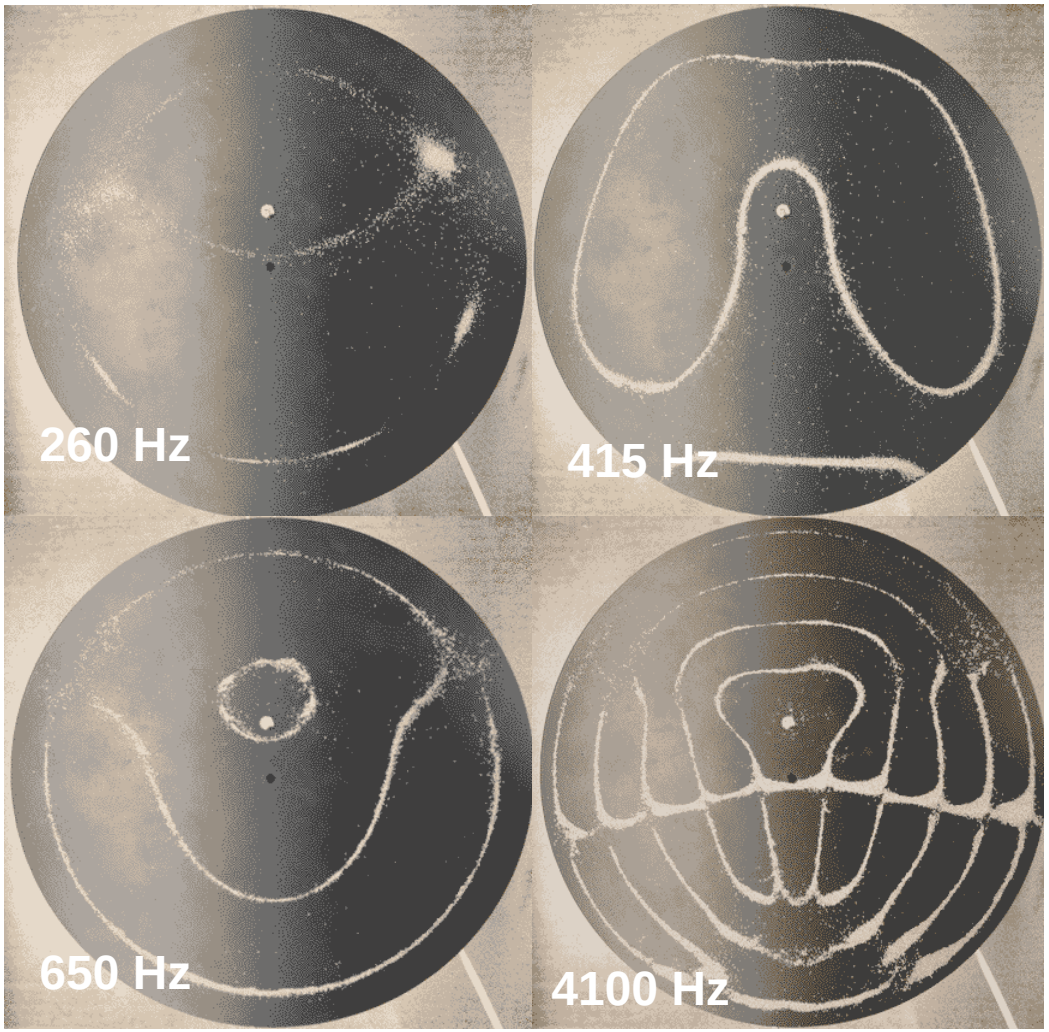
- For the circular plates we can verify chladni's law
  - Fairly simple since m = 0
  - So we plot frequency vs. $2n^2$ in Excel



We see a linear relationship between f and $2n^2$

Driving the Circular Plate Off Center

- Nodal Patterns are directly related to the driving location
  - Driving it off-center will produce different patterns!!

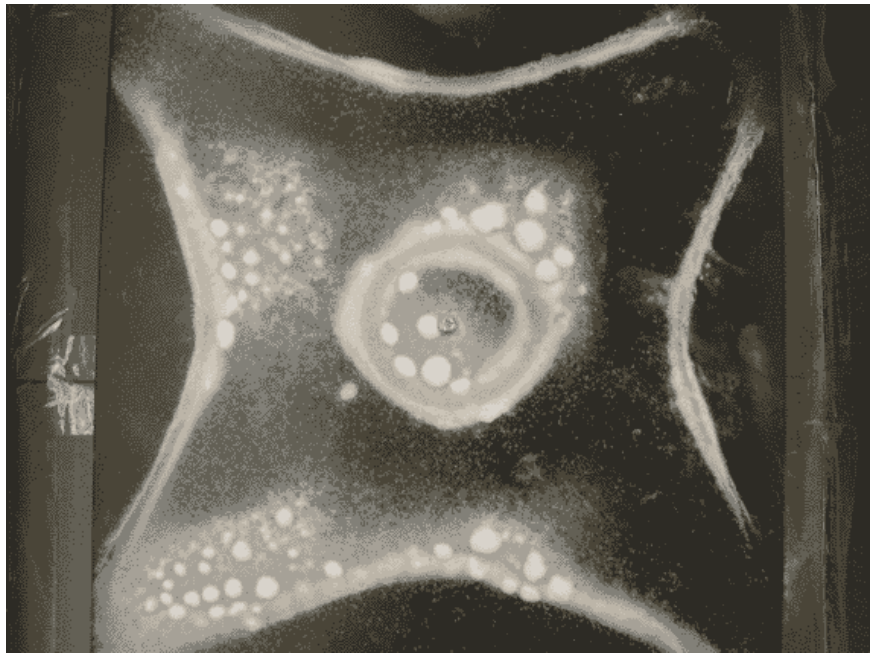


Material of Different Consistency

- We also tried a material (baking soda) of a finer consistency than sand
  - In theory this should produce different patterns
  - Why does this happen??
    - According to The Flying Circus of Physics..
    - “The fine dust [would also go to the nodes] except that it is carried by the air currents set in motion by the vibration. Since these currents blow across the plate from the nodes to the antinodes and then upward, the dust is carried to the antinodes and deposited.”
    - So, the dust reveals anti-nodal patterns…
    - But what actually happened?

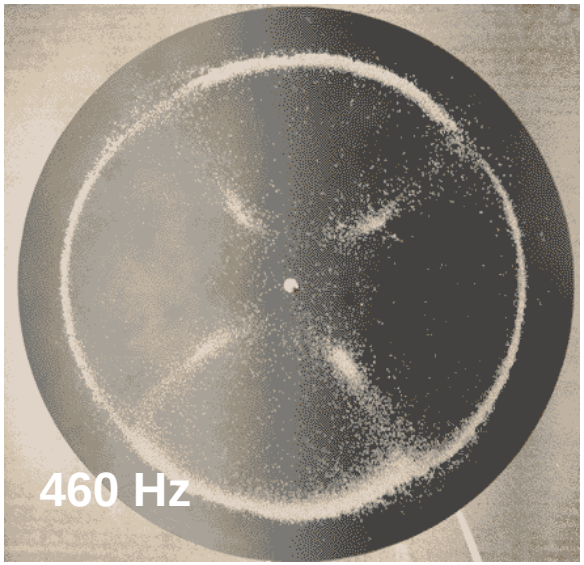
Well….

- We made a big mess..
- We did not see different patterns forming
  - Results may differ for a finer consistency material
- Baking soda clung to the plate temporarily
  - Probably due to static electricity
  - Formed clumps and moved to same nodes as sand moved to
  - Took longer to reach the nodes



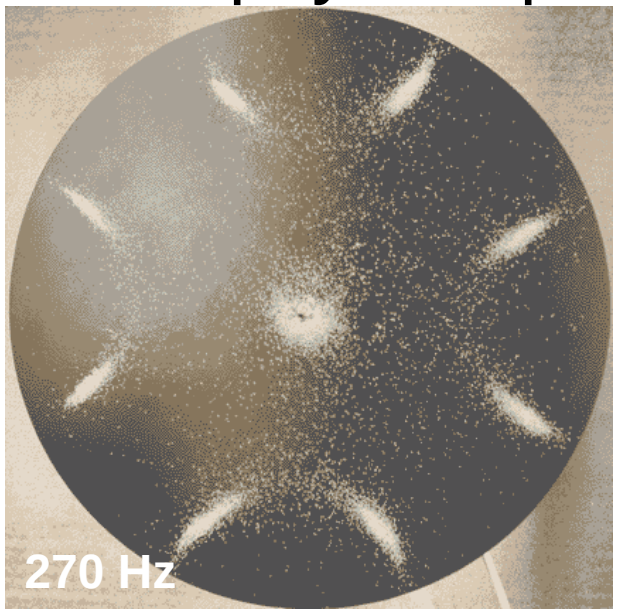
Strange Things

- We observed some strange things at various frequencies..
- At 470 Hz for the circular plate we noticed 2 diametric nodes beginning to form..
  - We had to increase the amplitude to see this pattern
  - Basically this makes the plate more sensitive..



Anomalies

- At one point we saw a pattern with symmetry that is unusual for circular plates
- This appears to be the four-fold symmetry we saw for the square plate
  - Not the radial symmetry we normally have for circular plates..why?
  - Probably due to physical properties of the plate



Symmetry

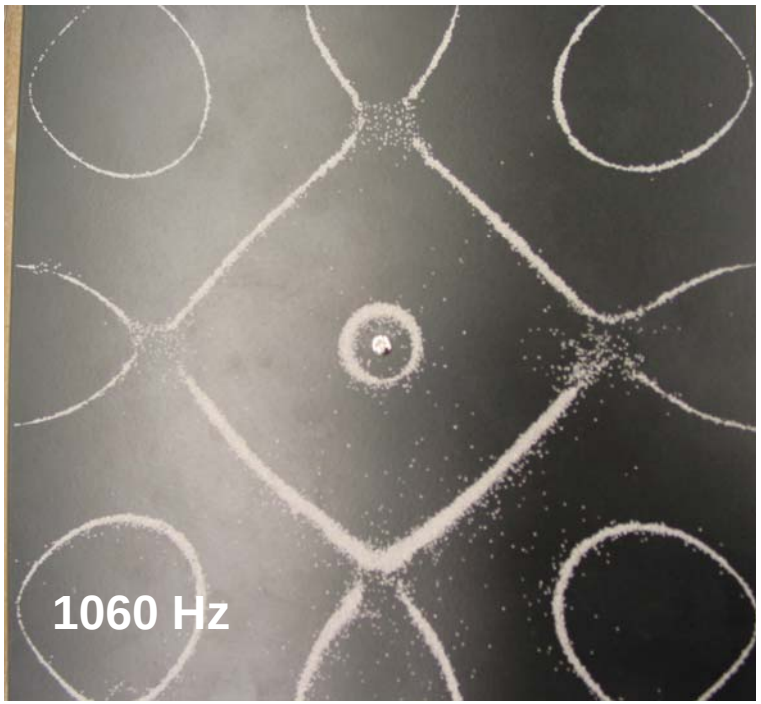
- For most cases we noticed a four-fold symmetry in the nodal patterns
- For several rare sets of frequencies (e.g. 390/395 Hz.) we see strange effects
  - Patterns do not exhibit four-fold symmetry
  - Patterns rotate 90^{o} for small frequency changes
  - Symmetry follows the plate
    - We hypothesize that these anomalies are caused by characteristics of the plate

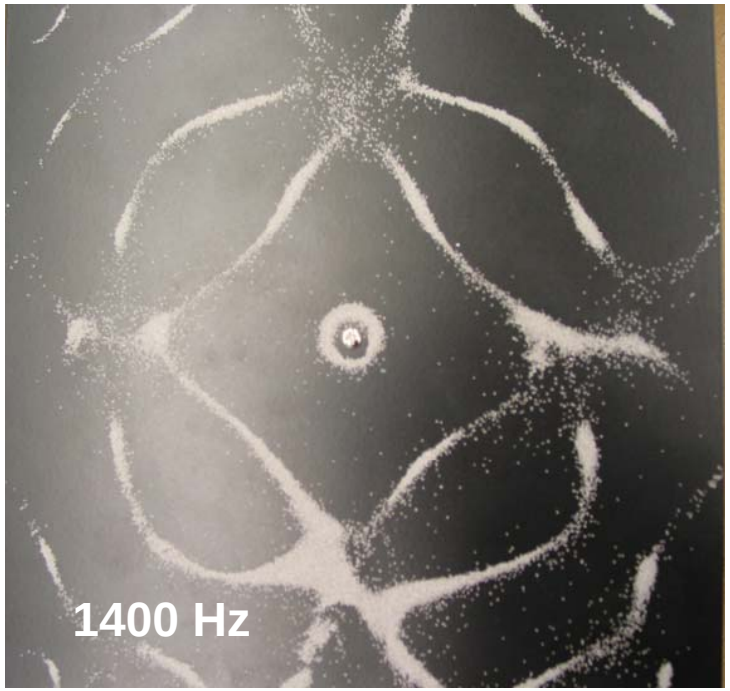
What else could we do?

- Nothing
  - Rather boring
- We could try to find the energy
  - If we new the amplitude then we could find the energy
  - Intensity is proportional to amplitude squared
  - Measure intensity at resonance
    - Use microphone and oscilloscope

Pretty Pictures!!

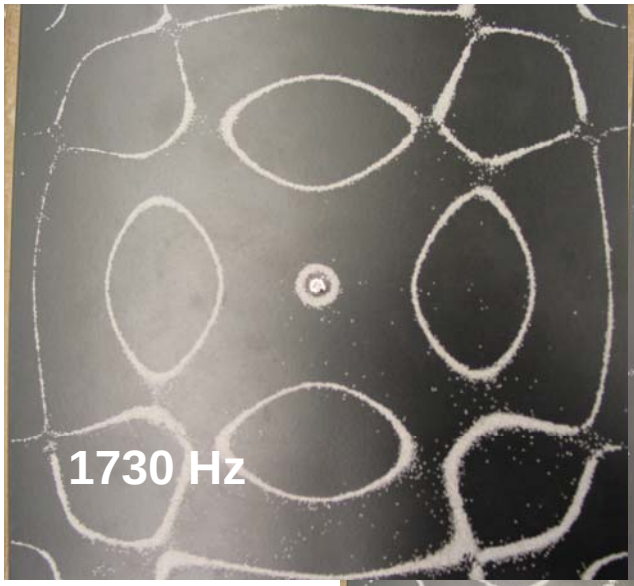
- Many of the patterns we observed for the square plate are difficult to explain
- But they look cool…

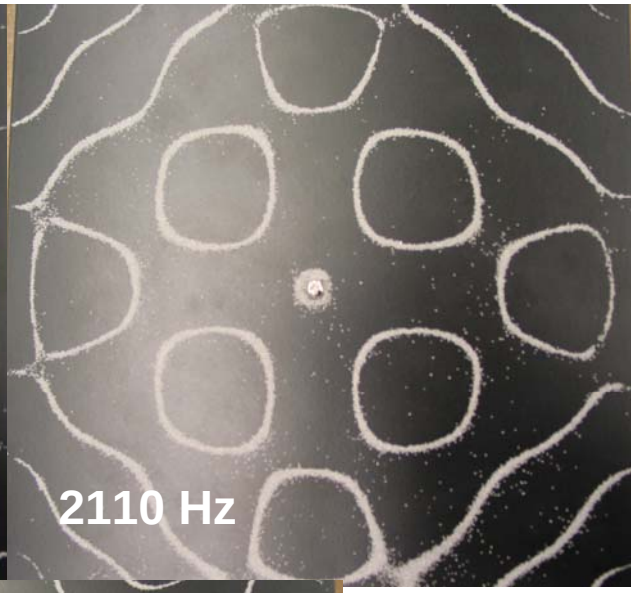




m?? n?? Who cares?!!

More…





Vibrational Holography

- Same patterns can be produced using holography
- 2 beams of coherent light interfere on holographic film
- Reference and object beam
  - Reference: fixed path length
  - Object: incident on vibrating plate
  - Vibration changes object beam path length
  - Causes interference rings around the antinodes

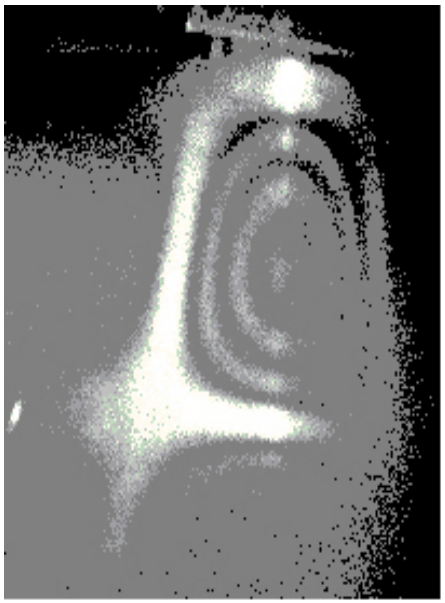


Figure 1-(3,1) mode being driven at –60.2 dB (relative)

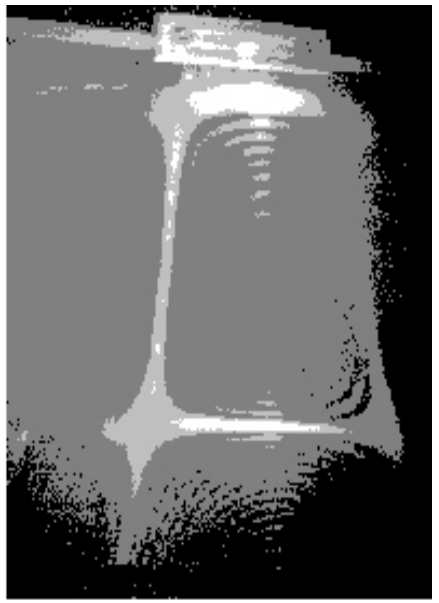


Figure 2-(3,1) mode driven at -50.0 dB (relative)

References

- http://www.stetson.edu/departments/physics/vholography/theory.htm
- http://www.rwc.uc.edu/koehler/biophys/9c.html
- http://astronomy.swin.edu.au/~pbourke/modelling/chladni/
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