MASSACHUSETTS INSTITUTE OF TECHNOLOGY
PHYSICS DEPARTMENT

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Junior Laboratory: Short Experiment #2

THE CAVENDISH EXPERIMENT
MEASUREMENT OF THE GRAVITATIONAL CONSTANT

PURPOSE

The purpose of this experiment is to measure G, the gravitational constant, by the method of Cavendish.

PREPARATORY PROBLEMS

1. Derive from first priciples the differential equation for damped, simple, harmonic motion. Derive the solution.
2. Compute the exact moment of inertia of two identical solid spheres of mass m and diameter d connected by a rod of mass μ and length l about an axis perpendicular to the rod.
3. Suppose the pendulum is at rest with the lead balls in rotated clockwise. Predict the curve of angular displacement *versus* time from the moment when the balls are rotated counterclockwise.

INTRODUCTION

According to Newton, two spherically symmetric bodies, A and B, with inertial masses, M_A and M_B, attract one another with a force of magnitude GM_AM_B/r^2 where r is the separation between the centers and G is the universal constant of gravity. The determination of G is obviously of fundamental importance in physics and astronomy. But gravity is the weakest of the forces, and the measurement of the gravity force between two bodies of measurable mass requires a delicate approach, with meticulous care to reduce perturbing influences such as air currents and electromagnetic forces. Henry Cavendish did it in 1798, a century after Newton's discovery of the law of universal gravitation. He used a torsion balance invented by one Rev. John Michell and, independently, by Charles Coulomb. Michell died shortly after completing his device, never having had the opportunity to apply it to the measurement of small forces for which he had devised it. It was passed on to Professor John Wollaston of Cambridge University and eventually to Cavendish who improved it and used it in a painstaking series of experiments to measure the mean density of the earth from which the value of G is readily derived. For the earth's mean density he found the value 5.48 g cm^3 with a stated uncertainty of 1 part in 14, which implies a value for G of $(6.70\pm0.48)\times10^{-8}$ dynes cm^2 g^{-2}. The current best value is $(6.67259\pm0.00085)\times10^{-8}$ dynes cm^2 g^{-2} . The large uncertainty, 128 ppm, compared to that of any of the other fundamental constants such as the elementary charge (0.30

ppm) and Planck's constant (0.60 ppm) reflects the fact that even today it is difficult to achieve an accurate measurement of G.

Our common experience with gravity is the weight of things. If A is an apple and E is the earth, then the weight of A is $W_{AE}=GM_AM_E/R_E^2=M_Ag$, where R_E is the radius of the earth and g is the acceleration of gravity. The latter two quantities can be measured easily to high accuracy (how?). Thus if you could measure M_E then you could determine G, or *vica versa*. It is clearly impossible to measure M_E as you do ordinary things, i.e. by direct comparison with a standard weight on a balance.

The only recourse is to replace the earth with a body B that can be measured directly, and to measure the force W_{AB} it exerts on the test body A. Suppose the radius of B is R_B and its density is ρ_B so that $M_B=(4\pi/3)R_B{}^3\rho_B$. Then if $r \approx R_B$, we find $W_{AB}=GM_A(4\pi/3)R_B\rho_B$. To get an idea of the practical difficulties that must be overcome in the measurement we can estimate the ratio of the force between a lead ball and a small test body at its surface to the force of earths gravity on the test body. The radius of the earth is $6.371\text{x}10^8$ cm. We can use Cavendish's value for the earth's mean density. If the radius of the lead ball is 3 cm and its density is 11.3 g cm^{-3}, then the ratio of forces is $W_{AB}/W_{AE}=(\rho_B/\rho_E)(R_B/R_E)\approx 10^{-8}$. Thus he had to measure a force on a test body that was about one hundred-millionth of its weight!

The torsion balance in the Junior Lab is shown schematically in Figure 1. It consists of a horizontal brass beam on the ends of which are two brass balls each of of mass m separated by a distance l between their centers, as shown in detail in Figure 2. The beam is suspended from its balance point by a fine tungsten wire which allows the beam to rotate about a vertical axis, subject to a restoring torque that is proportional to the angular displacement, θ, of the beam from its equilibrium orientation. The idea of the experiment is to measure the angular twist $\Delta\theta$ of the beam when two lead balls, each of mass M, are shifted from the positions labeled 1 to the positions labeled 2. If the distance between the center of each brass ball to the center of the nearest lead ball in both configuration 1 and 2 is called b, then the angular twist is

$$\Delta\theta = \frac{2GMml}{b^2\kappa} \tag{1}$$

where κ is the torsion constant. To measure this latter quantity we turn to the equation of motion of the torsion pendulum which is

$$\frac{d^2\theta}{dt^2} = -(\kappa/I)\theta - \beta\frac{d\theta}{dt}, \tag{2}$$

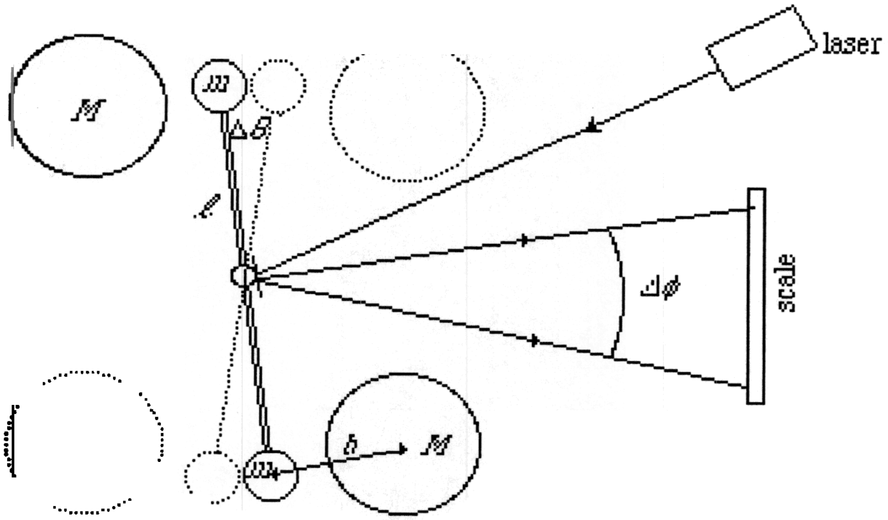

Figure 1. Schematic diagram of the torsion pendulum used in the Cavendish measurement of *G*.

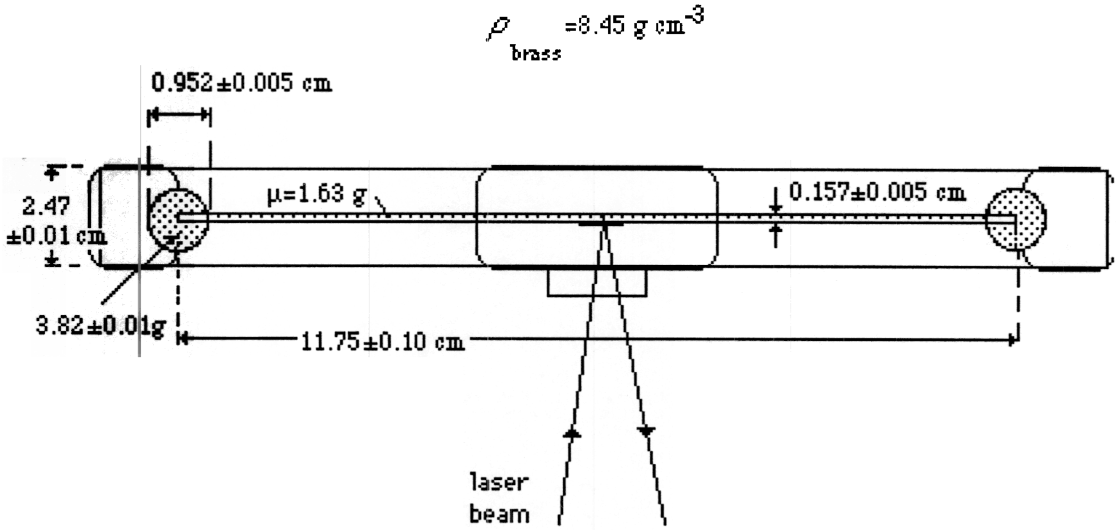

Figure 2. Details of the torsion balance beam, showing the two small brass balls mounted on the ends of a brass rod and suspended at the middle by a tungsten wire inside 1/2" pipe.

where I is the moment of inertia of the pendulum, and β is the coefficient of damping. With the initial condition $\theta(t=0)=0$ the solution to equation (2) is

$$\theta(t) = \theta_0 \exp(-\beta t/2)\sin(\omega t) \qquad (3)$$

where

$$\omega = \sqrt{\kappa/I - \beta^2/4}\ . \tag{4}$$

Equation (3) describes a damped harmonic motion about an equilibrium orientation with a period $T=2\pi/\omega$ and a characteristic damping time of $2/\beta$. In a typical setup this damping time is more than twice the period so that to a good approximation $\kappa=I\,(2\pi/T)^2$. If the beam is light compared to the brass balls, the moment of inertia is given to fair accuracy by the formula $I=ml^2/2$. Finally, one can determine the angular displacement caused by shifting the lead balls by measuring the angular deflection $\Delta\phi=2\Delta\theta$ of a laser beam reflected from a mirror mounted on the beam of the torsion pendulum. Substituting these quantities into equation (1) and rearranging we obtain an expression for G in terms of measurable quantities which is

$$G = \frac{b^2 l}{8M}\left(\frac{2\pi}{T}\right)^2 \Delta\phi \tag{5}$$

Note that the result is independent of the value of m.

The Junior Lab pendulum is suspended by a fragile 1 mil tungsten wire. The wire and beam are contained within copper plumbing to shield them from air currents and electric forces from stray static charges. Even the window for the laser beam is covered with fine wire mesh and glass. To avoid having to take the device apart at the risk of breaking the tungsten wire we provide you with the value of the distance between the brass balls , namely l= (11.75±0.10) cm. The other quantities are left for you to measure.

EXPERIMENT

Set up the laser so that its beam reflects from the mirror on the pendulum beam onto a meter stick mounted far enough away to facilitate an accurate measure of the angular displacement caused by shifting the lead balls. Ascertain whether the torsion pendulum is swinging freely about an equilibrium orientation near the center of its free range. If it isn't, make very gentle and cautious adjustment by twisting the fitting on the top of the pipe. (Take care not to snap the tungsten wire which is attached to a capstan in the top fitting. The capstan can be turned to raise or lower the pendulum. If it is raised too much, the wire will snap, which requires a long and tedious repair.) The pendulum can be gently maneuvered from the outside by the magnetic force exerted on the (paramagnetic) brass balls by a magnet. Center the rotating platform so that both lead balls touch, or come as close as possible to the brass pipes so they are at a well-determined position relative to the brass balls inside.

When you get the pendulum swinging freely with a very small amplitude about a central equilibrium position, start recording and plotting as you go the position of the reflected beam on the scale at regular intervals so that you have a record of the damped harmonic motion from which you can determine the period and damping time of the pendulum. After the amplitude has died to a small value or zero, shift the lead balls to the other position and begin regular periodic reading and plotting of the laser spot position on the scale. Go back and forth several times in this way to improve the statistical accuracy of your measurement.

Before shutting down check that you have measured all the relevant quantities.

ANALYSIS

Determine the period T and characteristic damping time τ (the time for the amplitude to decrease by 1/e). Compute the angular deflection from the displacement between the two equilibrium positions* of the laser spot on the scale. Compute the value of G from equation (5) with these and the other measured and given quantities. Estimate the random and systematic errors.

Using your value of G and the well known value of the acceleration of gravity at the earth's surface (which you can readily measure to high accuracy in a simple experiment), compute the mass of the earth. Using the period of the earth's orbit and the value of the astronomical unit compute the mass of the sun. And finally, given the period of the sun's orbit around the center of the galaxy (~2x10^8 yr) and its distance from the gactic center (~3x10^4 lt yr), estimate the mass of the galaxy.

Refinements

Consider the following corrections to the simple analysis above:

(1) the effect of damping on the pendulum period.

(2) the effect of the attractive forces between the lead balls and

  a) the opposite brass balls (weight=7.60 g),

  b) the brass beam (weight=1.625 g).

(3) error in the approximate calculation of the moment of inertia of the brass beam and ball.

(4) other things that come to mind.

*The equilibrium position of a lightly damped harmonic oscillator with three successive extreme displacements x_1, x_2, and x_3 is, to a very good approximation, $x_0=[(x_1+x_3)/2+x_2]/2$.

SUGGESTED THEORETICAL TOPICS FOR PRESENTATION AT THE ORAL REVIE

1. Corrections to the simple formula (equation 5).
2. Damped harmonic motion.
3. Limitations on the accuracy of Newton's Law of Gravity.