Physics 2514

Lecture 30

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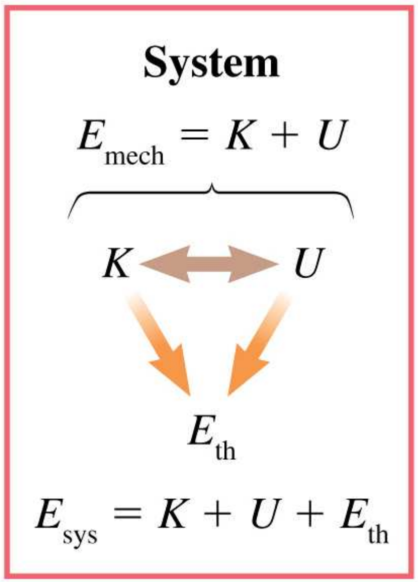
Goals

- We have introduced the concept of energy and energy conservation
  - We have dealt with mechanical energy, which corresponds to the kinetic and potential energies, only
- We still need to answer the following questions
  - How many kinds of energy are there?
  - Under what conditions is energy conserved?
  - How does a system lose or gain energy?
- We will introduce the concept of work to help with some of these questions

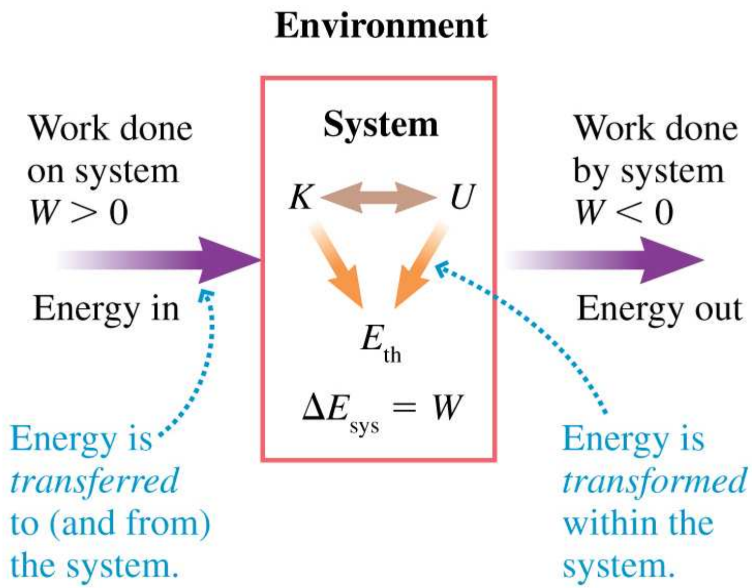
Energy Conservation

- $E_{\text{sys}} = (K + U + E_{\text{th}})$ and $\Delta E_{\text{sys}} = W$

Isolated System



Non-Isolated System



E_{th} thermal (internal) energy of the system, and W (work) is energy added or removed from the system.

Work

- Work: mechanical transfer of energy to or from a system
  - Due to forces
  - $W > 0$ work done on the system, energy increases;
    $W < 0$ work done by the system, energy decreases.

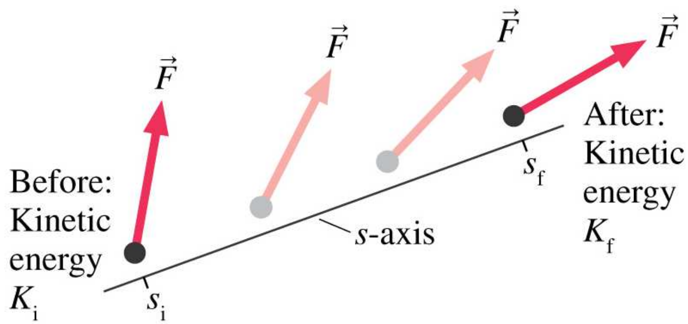
Newton's second law:

$$F_s = m\frac{dv_s}{dt} \quad \Rightarrow \quad F_s = mv_s\frac{dv_s}{ds}$$

$$\int_{s_i}^{s_f} F_s\, ds = \int_{v_i}^{v_f} mv_s\, dv_s$$

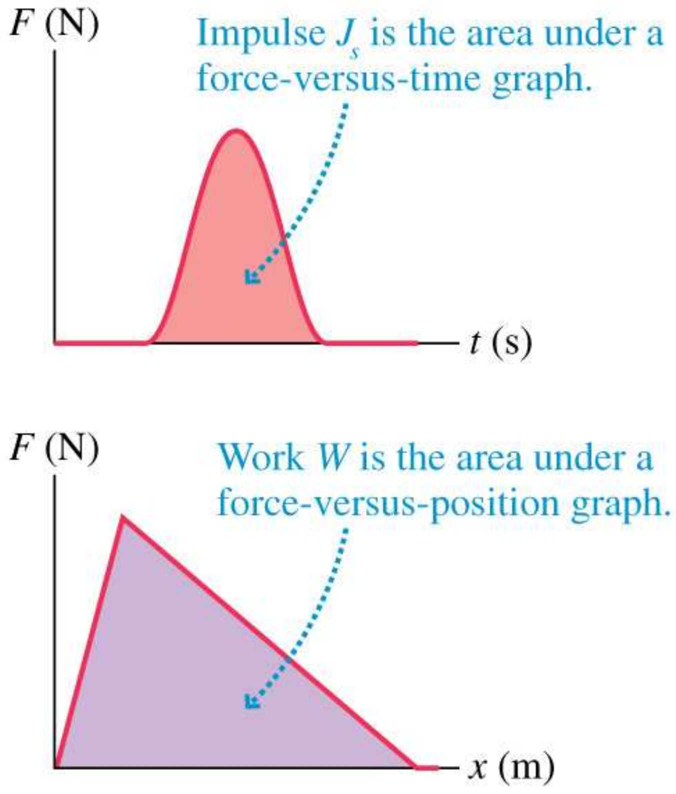
$$\Rightarrow \quad W = \Delta K$$

with: $W = \int_{s_i}^{s_f} F_s\, ds$



take only the component of the force along the direction of motion

Work & Impulse



Impulse-momentum theorem:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{J}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}\,dt$$

Work-kinetic energy theorem:

$$\Delta K = W = \int_{s_i}^{s_f} F_s\,ds$$

Clicker

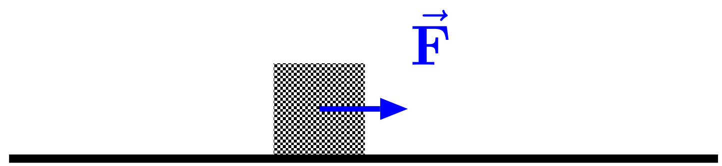
A 2 kg box on a flat frictionless horizontal surface has a constant 10 N horizontal force is applied to it over a distance of 5 m. The final velocity of the box is 7.1 m/s. How much work does the normal force do on the box.

A) 50 J

B) 35.5 J

C) 7.1 J

D) 0 J



Work

- Work-Kinetic Energy Theorem $(W = \Delta K)$
  - Work done by gravity

$$W = -\int_{y_i}^{y_f} mg\,dy = -(mgy_2 - mgy_1) = -\Delta U$$

  - Work done by a spring

$$W = -\int_{s_i}^{s_f} k\Delta s\,ds = -\left[\frac{1}{2}k(\Delta s_2)^2 - \frac{1}{2}k(\Delta s_1)^2\right] = -\Delta U$$

Example: A 10 kg object is dropped from rest. How much work is done by gravity if it drops 5 m? Assume object initially at $y_i = 5$ m

$$W = -\int_5^0 mg\,dy = -mg(0-5) = 490\ J$$

Clicker

A 1000 kg car travels 100 m up a mountain road at a constant speed of 30 m/s. What is the net work done on the car during the 100 m climb?

A) $0\ J$

B) $4.5 \times 10^5\ J$

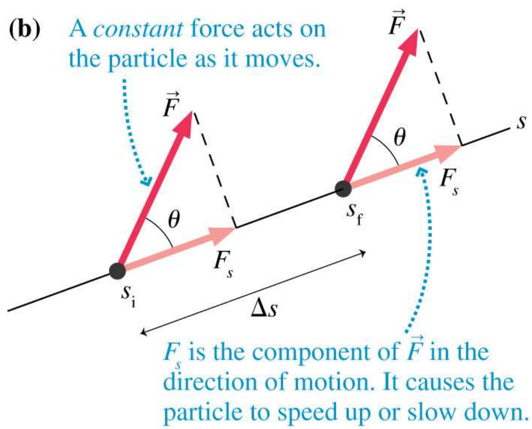
C) $9.8 \times 10^5\ J$

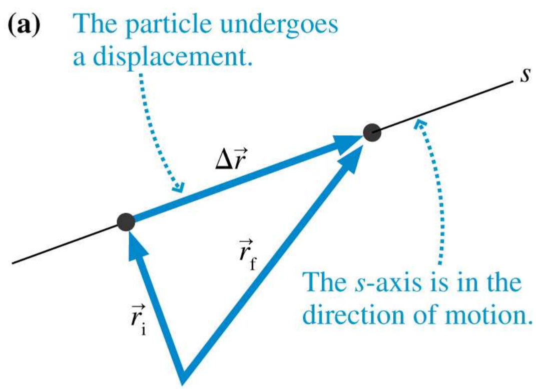
D) $1.4 \times 10^6\ J$

Work

- General expression for W (assuming F and θ constant)

$$W = \int_{s_i}^{s_f} F_s \, ds = \int_{s_i}^{s_f} F \cos\theta \, ds$$

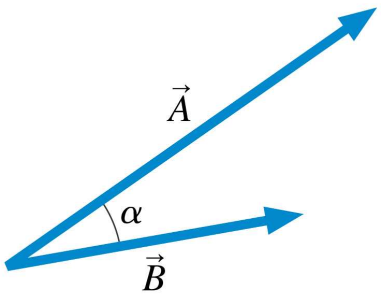




$W = F \cos\theta \, \Delta s = F \Delta s \, \cos\theta$ (Constant force, 1-d motion)

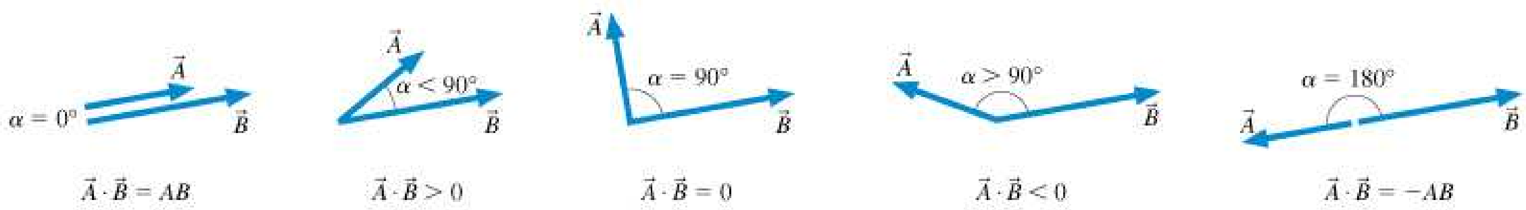
Dot Product

- Dot product of two vectors defined as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = |\vec{\mathbf{A}}||\vec{\mathbf{B}}| \cos\alpha$



Projection of $\vec{\mathbf{A}}$ on direction of $\vec{\mathbf{B}}$ is $|\vec{\mathbf{A}}| \cos\alpha$

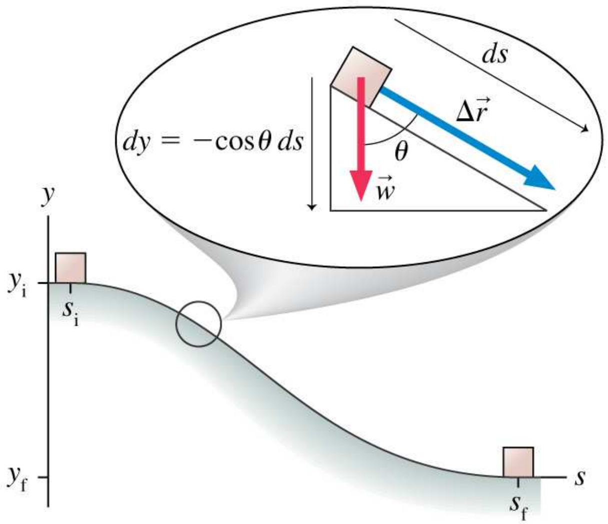
Projection of $\vec{\mathbf{B}}$ on direction of $\vec{\mathbf{A}}$ is $|\vec{\mathbf{B}}| \cos\alpha$



$$\left.\begin{aligned} \vec{\mathbf{A}} &= A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \\ \vec{\mathbf{B}} &= B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} \end{aligned}\right\} \quad \Rightarrow \quad \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y$$

Potential Energy

- Consider an object moving along an arbitrary path only acted on by the force of gravity



Work: $dW = m\vec{\mathbf{g}} \cdot d\vec{\mathbf{s}} = mg(\cos\theta\, ds)$

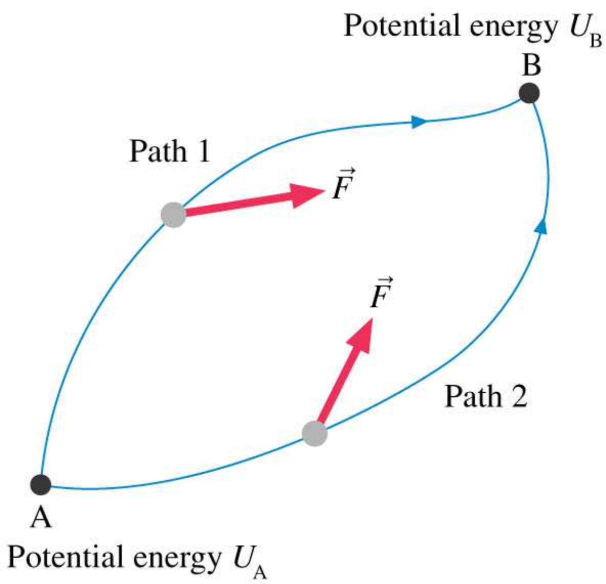
Note: $dy = -\cos\theta\, ds \Rightarrow W = -mg\Delta y$ (minus due to ds and dy having opposite sign by definition)

Therefore work done by gravity is independent of path, depends only on change in height

- If work is independent of path, force is called conservative, can be defined by a potential energy

Conservative Forces

- If work independent of path, force is conservative, and can be given in terms of a potential energy



If work path independent:

$$W = \Delta K_1 = \Delta K_2$$

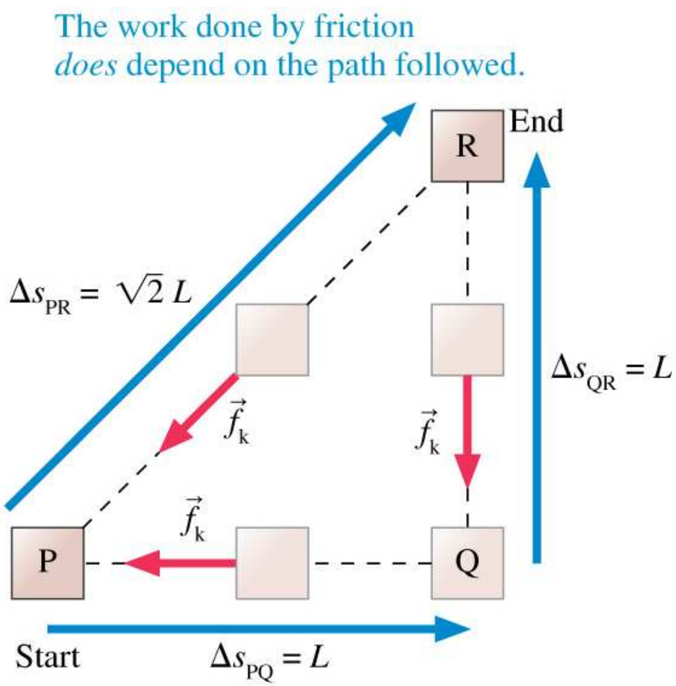
Then $W_1(A \rightarrow B) = -W_2(B \rightarrow A)$ (Force points in same direction independent of direction object moves)

Therefore work over closed loop is zero. This also holds for a spring

- This defines a conservative force

Friction

- Is friction a conservative force?
- Consider a box following two separate paths:



Work done by constant force:

$$W = \vec{\mathbf{f}}_k \cdot \Delta\vec{\mathbf{s}} = f_k \Delta s \cos(180)$$

$$W = -mg\mu_k \Delta s$$

Path $P \rightarrow R$: $W = -\sqrt{2}mg\mu_k L$

Path $P \rightarrow Q \rightarrow R$: $W = -2mg\mu_k L$

Work depends on path

Friction

- Friction is a nonconservative force, can not be written in the form of a potential energy
  - Friction cannot store energy
- Work in general can be written as the sum of a piece that is conserved and a piece that isn't $W_{\text{net}} = W_c + W_{nc} = \Delta K$
- Change in the mechanical energy equals work done by nonconservative forces $\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}$ (where $W_c = -\Delta U$ was used)
  - The nonconservative work is the sum of dissipative forces (such as friction) and external forces $W_{nc} = W_{diss} + E_{ext}$
  - Thermal energy gained by system is $\Delta E_{th} = -W_{diss}$

Assignment

Continue reading chapter 11