Physics 4213/5213
Lecture 18

1 Introduction

At this point all the pieces have been developed to construct the electroweak model. These pieces include the development of non-Abelian gauge theories and the development of spontaneous symmetry breaking for the generation of gauge boson masses in a gauge invariant way. It will be seen in this lecture, that spontaneous symmetry breaking will also lead to the generation of fermion masses.

To start with the particles that make up the standard model of particle physics are the six leptons, particles that do not exhibit strong interactions and the six quarks that do see all interactions. All these particles are assumed to be point like (they have no discernible structure) and they are all Fermions. The leptons have an absolutely conserved quantum number (family lepton number) that does not allow them to transform between families; this is due to the neutrinos being massless. The quarks, through the weak interaction are allowed to change flavor directly, but the total number of quarks is conserved (baryon number conservation).

2 Selection of the Gauge Group

The goal of selecting a gauge group, will be to relate weak and electromagnetic interactions to each other; for now, without loss of generality, the model will be developed for leptons only, quarks will be introduced later. To do so, start by looking at the basic structure of the weak interaction theory developed so far and QED. Each of the two theories are composed of a current that couples to a vector boson ($l \to l+\gamma$, $\nu_l \to l^- + W^+$ and $l^- \to \nu_l + W^-$. The electromagnetic current is given by

$$j^\mu_{em} = \bar{u}\gamma^\mu u \tag{1}$$

where u represents any point-like electrically charged particle; note that anti-particles can also be included by replacing u with v. On the other hand, the weak interaction current is given by

$$j^\mu_w = \bar{u}_{\nu_l}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u_l \tag{2}$$

where a change in charge between incoming and outgoing particles is noted. In addition, a weak neutral current can also be introduced, but since it hasn't been introduced until now, it will be allowed to appear naturally later on.

To relate the two currents to each other, they should be written in a similar form. At present the electromagnetic current is pure vector while the weak current involves both a vector and an axial vector component. To begin the process of making the two currents look the same, take note of the following two relations

$$(1-\gamma^5)(1-\gamma^5) = 2(1-\gamma^5) \tag{3}$$

$$\gamma^\mu(1-\gamma^5) = (1+\gamma^5)\gamma^\mu. \tag{4}$$

Now define chirality states as given in table 1. With these definitions and equations 3 and 4, the weak currents can be written as

$$j_w^\mu = \bar{u}_\nu \gamma^\mu \frac{(1-\gamma^5)}{2}\frac{(1-\gamma^5)}{2} u_l = \bar{u}_\nu \frac{(1+\gamma^5)}{2}\gamma^\mu \frac{(1-\gamma^5)}{2} u_l = \bar{u}_{\nu L}\gamma^\mu u_{lL} \tag{5}$$

where the current has now been written in the form of a vector current. This almost puts it on an equal footing as the electromagnetic current. The only difference is that only states of definite chirality come into play.

Particles	Anti-Particles
$2u_L = (1-\gamma^5)u$	$2v_L = (1+\gamma^5)v$
$2u_R = (1+\gamma^5)u$	$2v_R = (1-\gamma^5)v$
$2\bar{u}_L = \bar{u}(1+\gamma^5)$	$2\bar{v}_L = \bar{v}(1-\gamma^5)$
$2\bar{u}_R = \bar{u}(1-\gamma^5)$	$2\bar{v}_R = \bar{v}(1+\gamma^5)$

Table 1: The left and right chiral states for the various Dirac spinors.

If the electromagnetic current can be put into a form that only involves the states of definite chirality, then an equivalence can be made between the two interactions. First of all, write the spinor u as

$$u = (1-\gamma^5)u + (1+\gamma^5)u = u_L + u_R. \tag{6}$$

The electromagnetic current then becomes

$$j_{em}^\mu = [\bar{u}_L + \bar{u}_R]\gamma^\mu[u_L + u_R] = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R \tag{7}$$

where the final expression is derived by using equations 3 and 4. This now puts the two interactions on an equal footing, in that they are both vector currents and involve states of definite chirality.

Now there are two types of weak currents, those that cause a change of charge by +1 from initial to final lepton and those that cause a change of charge of −1. These two currents are

$$j_\mu^- = \bar{u}_{\nu L}\gamma_\mu u_{lL} \qquad\qquad j_\mu^+ = \bar{u}_{lL}\gamma_\mu u_{\nu L}. \tag{8}$$

These two equations can then be written in a simple compact form

$$j_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L \tag{9}$$

where

$$\chi_L = \begin{pmatrix} u_\nu \\ u_l \end{pmatrix}_L \qquad \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{10}$$

The two $\tau^\pm$ matrices are composed of two of the Pauli spin matrices

$$\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2); \tag{11}$$

note these are two of the generators of the group $SU(2)$. Therefore the left handed chiral states have been put into an $SU(2)$ doublet state. Given that there is no right hand neutrino, the right handed massive lepton must go into an $SU(2)$ singlet state.

If the structure found so far is $SU(2)$, then it can be argued that a neutral component to the current must also exist corresponding to the third generator of the group. This current would look like

$$j^3_\mu = \bar{\chi}_L \gamma_\mu \frac{1}{2}\tau^3 \chi_L = \frac{1}{2}\bar{u}_{\nu L}\gamma_\mu u_{\nu L} - \frac{1}{2}\bar{u}_{lL}\gamma_\mu u_{lL}. \tag{12}$$

The only problem with this is, that it is now known that the weak neutral current couples both left and right handed fermions together. Further, in an attempt to get electromagnetic interactions into this model, the neutral current will be made to include the electromagnetic current. This can be done in the following manner. Define a hypercharge that connects the electric charge and the third component of weak isospin

$$Q = I_3 + \frac{1}{2}Y; \tag{13}$$

as was done in for the original strong interaction isospin. The hypercharge current can be written as

$$j^Y_\mu = 2j^{em}_\mu - 2j^3_\mu = -2\bar{u}_{lR}\gamma_\mu u_{lR} - \bar{u}_{lL}\gamma_\mu u_{lL} - \bar{u}_{\nu L}\gamma_\mu u_{\nu L}. \tag{14}$$

Thus there appear to be two symmetry groups involved, one that handles the left handed doublet states $SU(2)$ and one that handles both left and right handed particle states through the hypercharge $U(1)$. The gauge group that will be used to build the electroweak model from is therefore the product group $SU(2) \otimes U(1)$.

3 The Electroweak Lagrangian

Given the gauge group for the theory, the Lagrangian can now be developed. To start with, it will be composed of the following terms

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_s + \mathcal{L}_{f-s}. \tag{15}$$

It will also have two coupling constants, one for each of the gauge groups; the $SU(2)$ coupling constant is given by g while the $U(1)$ coupling constant is $g'/2$.

The first term ($\mathcal{L}_g$) is the term that has only the gauge fields in it and is given by

$$\mathcal{L}_g = -\frac{1}{4}G^{\mu\nu}_i G^i_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{16}$$

where

$$G^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu \tag{17}$$

and

$$F^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{18}$$

Also keep in mind that the $G_i^{\mu\nu}G^i_{\mu\nu}$ term includes pieces that couple the gauge fields together. The next term contains the Fermions. It is given by

$$\begin{aligned}\mathcal{L}_f &= i\bar{u}_R\gamma^\mu D_\mu u_R + i\bar{\chi}_L\gamma^\mu D_\mu \chi_L \\ \mathcal{L}_f &= i\bar{u}_R\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu y\right)u_R + i\bar{\chi}_L\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu y + i\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu\right)\chi_L\end{aligned} \tag{19}$$

This piece describes the free Fermion fields plus their interaction with the gauge fields.

The next term in the Lagrangian, introduces the scalar fields that will be used to break the symmetry of the vacuum. This term will be used to break the symmetry of the $SU(2)$ portion of the gauge group allowing the three vector fields to gain a mass, but preserve the vacuum for $U_{em}(1)$ to maintain a massless photon; note the $U(1)$ portion of the original gauge group corresponds to hypercharge so that some mixing will have to occur to get the electromagnetic portion. To break the vacuum of $SU(2)$ part of the gauge group introduce a doublet of scalar fields

$$\phi \equiv \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \tag{20}$$

with a hypercharge of +1—the reason for this form of the doublet, is that the lower component will couple the spinors associated with the charged leptons and will therefore be connected with the vacuum expectation value which must produce a neutral field. The Lagrangian for the scalar fields is

$$\mathcal{L}_s = (D^\mu\phi)^\dagger(D_\mu\phi) - \mu^2(\phi^\dagger\phi)\lambda(\phi^\dagger\phi)^2. \tag{21}$$

Finally, a term that couples the scalar fields to the Fermions is added

$$\mathcal{L}_{s-f} = -G[\bar{u}_R\phi^\dagger\chi_L + \bar{\chi}_L\phi u_R]; \tag{22}$$

notice that this couples left handed terms (doublets) to right handed terms, so that there will be no such term for the neutrino only for the massive particles.

4 Spontaneous Symmetry Breaking

With the Lagrangian in hand, the symmetry of the vacuum can be broken by applying the condition $\mu^2 < 0$ as was done in the previous lecture. The condition that is desired is that the vacuum break both the $SU_L(2)$ symmetry and the $U_Y(1)$ symmetry but not the $U_{em}(1)$ symmetry. This can be achieved by selecting the vacuum expectation value to equal

$$\langle\phi\rangle_0 = \begin{pmatrix}0 \\ v/\sqrt{2}\end{pmatrix}. \tag{23}$$

To show that this does break the symmetry, it needs to be shown that

$$e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 \neq \langle\phi\rangle_0, \tag{24}$$

where $\mathcal{G}$ is equal to the generators of the group. Since both $SU(2)$ and $U(1)$ are Lie groups, the same condition must hold for infinitesimal transformations from the identity. This condition implies

$$(1 + i\alpha\mathcal{G})\langle\phi\rangle_0 \neq \langle\phi\rangle_0, \tag{25}$$

or that

$$\mathcal{G}\langle\phi\rangle_0 \neq 0. \tag{26}$$

Applying the generators to the vacuum expectation value gives

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \tag{27}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \tag{28}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \tag{29}$$

$$Y\langle\phi\rangle_0 = +1\langle\phi\rangle_0 \neq 0 \tag{30}$$

$$\tag{31}$$

but the charge generator acting on the vacuum expectation value gives

$$Q\langle\phi\rangle_0 = \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = 0. \tag{32}$$

Therefore, the vacuum expectation value of the scalar field is invariant to the charge operator (electromagnetism) but not to $SU_L(2)$ and $U_Y(1)$, that is after symmetry breaking the group becomes symmetric only under the electromagnetic interaction ($SU_L(2) \otimes U_Y(1) \rightarrow U_em(1)$). This is exactly what was being sought, a theory with massive weak bosons and a massless photon. The next step is to show that the breaking of the vacuum symmetry leads to massive particles.

5 The Generation of Masses

In the last lecture, the symmetry of the vacuum was spontaneously broken for both the $SU_L(2)$ and $U_Y(1)$ gauge groups, but it was preserved for the $U_{em}(1)$ gauge group. This will allow three of the gauge bosons to acquire masses while still leaving one massless; this is due to the breaking of the symmetry for only three of the generators, since $U_{em}(1)$ is only a rotation of $U_Y(1)$. To determine the mass spectrum, the Lagrangian is expanded for small deviations of the shifted fields about the new vacuum. The shifted field is given by

$$\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix} e^{\frac{i\boldsymbol{\xi}\cdot\boldsymbol{\tau}}{2v}}. \tag{33}$$

To show explicitly that the Goldstone bosons are eaten by the gauge fields, the U-gauge is used. Applied to the field this gives

$$\phi \rightarrow \phi e^{\frac{-i\boldsymbol{\xi}\cdot\boldsymbol{\tau}}{2v}} = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}. \tag{34}$$

The remaining terms must then transform as

$$\boldsymbol{\tau}\cdot\mathbf{W}^\mu \rightarrow \boldsymbol{\tau}\cdot\mathbf{W}'^\mu \qquad B^\mu \rightarrow B^\mu$$

$$u_R \rightarrow u_R \qquad \chi_L \rightarrow e^{\frac{-i\boldsymbol{\xi}\cdot\boldsymbol{\tau}}{2v}}\chi_L, \tag{35}$$

where

$$\mathbf{W'}_\mu = U\mathbf{W}_\mu U^{-1} - \frac{i}{g} U\partial_\mu U^{-1}, \qquad U = e^{\frac{-i\boldsymbol{\xi}\cdot\boldsymbol{\tau}}{2v}} \tag{36}$$

which for an infinitesmal transformation is given by:

$$\mathbf{W}'_\mu = \mathbf{W}_\mu - \frac{1}{g}\partial_\mu \boldsymbol{\xi} - \boldsymbol{\xi} \times \mathbf{W}_\mu \tag{37}$$

With the vacuum selected in the previous lecture and using the U-gauge, each term in the Lagrangian can be examined in turn. The first term will be the $\mathcal{L}_{f-s}$.

5.1 The Fermion Masses

As a reminder, the term in the Lagrangian that couples scalars and Fermions together is given by

$$\mathcal{L}_{f-s} = -G[\bar{u}_R \phi^\dagger \chi_L + \chi_L \phi u_R]. \tag{38}$$

To explore the mass spectrum generated by the symmetry breaking of the Higgs potential, the fields in the U-gauge given in equations 34 and 35 are replaced in the Lagrangian. This gives

$$\mathcal{L}_{f-s} = -G\frac{v+\eta}{\sqrt{2}}(\bar{e}_R e_L + \bar{e}_L e_R) = \frac{-Gv}{\sqrt{2}}\bar{e}e + \frac{-G}{\sqrt{2}}\eta\bar{e}e, \tag{39}$$

which uses the relation:

$$\frac{1}{2}(1-\gamma^5)\frac{1}{2}(1-\gamma^5) = \frac{1}{2}(1-\gamma^5) \tag{40}$$

and the mass of the Fermion is given by

$$m = \frac{Gv}{\sqrt{2}} \tag{41}$$

The first term involves only the electron fields (left and right coupled together) and therefore must correspond to the a mass term; notice that electron refers to all massive Fermions. The second term in the Lagrangian, gives the coupling between the Fermion field and the Higgs field. Therefore the Fermion Higgs vertex term is given by

$$\Gamma(e,\eta) = -i\frac{G}{\sqrt{2}} = -i\frac{m}{v} = -im\sqrt{G_F\sqrt{2}} \tag{42}$$

where it is noted that the Higgs-Fermion coupling depends on the mass of the Fermion.

5.2 The Gauge Boson Masses

The starting point to explore the masses of the gauge bosons is the the Lagrangian for the scalar fields. All other terms simply describe how the gauge fields interact, it is the Lagrangian of the scalar field that is is responsible for the gauge fields acquiring mass through the Higgs mechanism. As a reminder the scalar field Lagrangian is given by

$$\mathcal{L}_s = (D^\mu\phi)^\dagger(D_\mu\phi) - \mu^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2. \tag{43}$$

where

$$D^\mu = \partial^\mu + i\frac{g'}{2}B^\mu y + i\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}^\mu \tag{44}$$

and $y = -1$ for the (particle) lepton doublets. As before, the mass spectrum is determined by looking at the shifted field about the new vacuum state. This is done by substituting equations 34 and 35 into equation 43

$$\mathcal{L} = \frac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \frac{1}{2}m_\mu^2\eta^2 + \frac{1}{2}m_W^2\left(|W_\mu^+|^2 + |W_\mu^-|^2\right) + \frac{1}{2}m_Z^2|Z_\mu^0|^2 + \text{interaction terms} \tag{45}$$

where the masses are given by

$$m_W = \frac{gv}{2} \qquad\qquad m_Z = m_W\sqrt{\frac{g^2+g'^2}{g^2}} \tag{46}$$

and the fields are given by

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \qquad\qquad Z_\mu^0 = \frac{-g'B_\mu + gW_\mu^3}{\sqrt{g^2+g'^2}}. \tag{47}$$

One final field needs to be defined, this is the field that is orthogonal to the Z^0 field which is given by

$$A_\mu = \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2+g'^2}} \tag{48}$$

because this term does not occur in the scalar Lagrangian, it corresponds to a massless term, since no other piece of the Lagrangian introduces massive gauge fields. This last field corresponds to the photon.

The only remaining mass term that has not been examined in the scalar Lagrangian is $m_\eta^2\eta^2$. This corresponds to the now massive scalar Higgs boson. Its mass is given by

$$m_\eta = \sqrt{-2\mu^2} \tag{49}$$

and can not be written in terms of other known parameters.

6 Interaction of Fermions and Gauge Bosons

Now that three of the original four gauge bosons have acquired a masses, how they interact with the Fermions of the theory needs to be examined. To do so, the Fermion term $\mathcal{L}_f$ is examined, since it is this term that contains all the information on how Fermions and gauge bosons interact. This term is given by

$$\mathcal{L}_f = i\bar{u}_R\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu y\right)u_R + i\bar{\chi}_L\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu y + i\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu\right)\chi_L, \tag{50}$$

where the first term takes into account that the right handed terms have $I = 0$, further as stated earlier $y = -1$ for the isodoublet and $y = -2$ for the right-handed isosinglet. To determine how the Fermions interact with the massive gauge bosons ($W^{\pm}$ and Z^0) and the one remaining massless gauge boson, the fields in the Lagrangian (equation 50) are put into the form given in equations 47 and 48. After factoring out the fields in the Lagrangian, the charge current has the following form

$$\begin{aligned}\mathcal{L}_{W-l} &= -\frac{g}{\sqrt{2}}(\bar{\nu}_L\gamma^\mu e_L W_\mu^+ + \bar{e}_L\gamma^\mu\nu_L W_\mu^-) \\ &= -\frac{g}{2\sqrt{2}}(\bar{\nu}\gamma^\mu(1-\gamma^5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma^5)\nu W_\mu^-)\end{aligned} \tag{51}$$

where for definiteness, the electron and neutrino fields are used. As would be expected, this is just the effective Lagrangian that the Fermi theory predicts for a massive vector boson. The Feynman rules for the vertices of this Lagrangian are given by

$$\begin{aligned}&e^- \to \nu + W^- \qquad && -i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \\ &\nu \to e^- + W^+ \qquad && -i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\end{aligned} \tag{52}$$

where the expressions for the vertices given in the phemenological model can be recovered using the identity:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \tag{53}$$

and moving the M_W^2 term over to the propogator.

The neutral portion of the Lagrangian can be derived in the same fashion, by taking the field terms in equations 47 and 48, then factoring these terms out of the Lagrangian in equation 50. This leads to the neutral portion of the Lagrangian being

$$\begin{aligned}\mathcal{L}_{0-l} = &\frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu eA^\mu - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z^\mu \\ &+ \frac{Z^\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{\sqrt{g^2-g'^2}}{2}\bar{e}_L\gamma^\mu e_L\right].\end{aligned} \tag{54}$$

Notice that for the massive Fermion field, there are terms that couple both left and right handed fermions to the Z^0. This is to be compared to the charged vector bosons, where only left handed Fermions coupled to the $W^{\pm}$.

To make the assignment of the A^μ field to the photon, the coupling constant associated with it must be the electric charge

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}. \tag{55}$$

Further, a convenient parameterization of the coupling constants is given by

$$g' = \frac{g}{\tan\theta_W} \Longrightarrow \sqrt{g^2+g'^2} = \frac{g}{\cos\theta_W} \Longrightarrow e = g\sin\theta_W, \tag{56}$$

keep in mind that the g and g' are the coupling constants of the original unbroken symmetries. The angle θ_W is called the Weinberg angle. This also states that the electromagnetic coupling constant is arrived at by a simple rotation in Y-I_3 space, as should be expected since the electromagnetic field is obtained by mixing the hypercharge with the third component of weak isospin. One other point to notice, is that the coupling constants for weak and electromagnetic interactions are of the same order of magnitude depending on the value θ_W, but given that $\sin^2\theta_W \approx 0.23$ the approximate equality of the two coupling constant is shown. From these relations the two massive gauge boson masses are given by

$$M_W^2 = \frac{e^2}{4\sqrt{2}\,G_F \sin^2\theta_W} = \frac{(37.3\mathrm{GeV})^2}{\sin^2\theta_W} \qquad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \Longrightarrow M_Z \geq M_W \tag{57}$$

The current measured values of the masses are:

$$M_W = 80.44 \pm 0.1 \pm 0.007\,\mathrm{GeV} \qquad M_Z = 91.187 \pm 0.007\,\mathrm{GeV} \tag{58}$$

Finally the value of the vacuum expectation for the broken symmetry is given by:

$$\frac{v}{\sqrt{2}} = \left(\sqrt{8}G_F\right) \approx 174\,\mathrm{GeV} \tag{59}$$

To make the mixing of the fields more obvious, the fields can be written in terms of θ_W as

$$\begin{aligned} Z_\mu &= -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \\ A_\mu &= B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W. \end{aligned} \tag{60}$$

This lead to the Lagrangian in terms of the Weinberg angle as

$$\begin{aligned} \mathcal{L}_{0-l} = e\bar{e}\gamma^\mu e A^\mu &- \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma^\mu(1-\gamma^5)\nu Z^\mu \\ &- \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \left[2\sin^2\theta_W \bar{e}\gamma^\mu(1+\gamma^5)e + (2\sin^2\theta_W - 1)\bar{e}\gamma^\mu(1-\gamma^5)e\right] Z_\mu. \end{aligned} \tag{61}$$

Finally, from both the charge and neutral terms in the Lagrangian, the vertex factors are extract below.

$$\begin{array}{ll} e \to e + \gamma & ie\bar{e}\gamma_\mu e \\ e \to \nu + W^- & -i\left(\dfrac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma^5)e \\ \nu \to \nu + Z^0 & -\dfrac{i}{\sqrt{2}}\left(\dfrac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma^5)\nu \\ e \to e + Z^0 & -\dfrac{i}{\sqrt{2}}\left(\dfrac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{e}\left[\gamma_\mu(2\sin^2\theta_W)(1+\gamma^5) + (2\sin^2\theta_W - 1)(1-\gamma^5)\right] e \end{array} \tag{62}$$

7 Problems

1. Derive equation 45 including the interaction terms.
2. Show that the definition of the photon in equation 48 is the correct. *Start with the gauge term in the Lagrangian and the definition of the Z^0 field.*
3. Derive equations 51 and 54 starting with equation 50.