Physics 6213
Homework 4

1 Problem

The following problem will be due Mon April 19, 1999. Write up the solution and hand it in at the beginning of class. Everyone should be prepared to discuss this problem in class.

1. Muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ is described by the invariant amplitude:

$$\mathcal{M} = \left(\frac{g}{\sqrt{2}}\bar{u}_{\nu_\mu}\gamma^\sigma \frac{1}{2}(1-\gamma^5)u_\mu\right)\frac{1}{M_W^2 - q^2}\left(\frac{g}{\sqrt{2}}\bar{u}_e\gamma_\sigma\frac{1}{2}(1-\gamma^5)v_{\nu_e}\right)$$

where $g/\sqrt{2}$ is the dimensionless weak coupling constant, and q^2 is the square of the 4-momentum transfer.

(a) Explain why the invariant amplitude can be written in terms of a single coupling constant with the dimensions of an inverse mass $G_F/\sqrt{2} = g^2/8M_W^2$ times a current-current interactions for muon decay. *This is referred to as a four-point interaction.*

(b) Using the form of the invariant amplitude with the Fermi coupling constant $G_F/\sqrt{2}$, derive the differential decay rate:

$$\frac{d\Gamma}{dE_e} = \frac{G_F m_\mu^2}{4\pi^3}\left(1 - \frac{4E_e}{3m_\mu}\right)$$

(c) Finally, derive the total decay rate:

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$