Physics 6213
Homework 3

1 Problem

The following problem will be due Mon Feb. 22, 1999. Write the solution out and somebody should be prepared to discuss in class.

1. As a reminder, the current for a spin zero particle is given by:

$$j^{\mu}_{fi} = -e(p_f + p_i)^{\mu} e^{i(p_f - p_i)^{\mu} x_{\mu}}$$

notice that this current interacts $j^{\mu}_{if} A_{\mu}$ only through its charge. Show that the Dirac current can be expressed as:

$$\bar{u}_f \gamma^{\mu} u_i = \frac{1}{2m} \bar{u}_f \left[(p_f + p_i)^{\mu} + i\sigma^{\mu\nu}(p_f - p_i)_{\nu} \right] u_i$$

this is known as the Gordon decomposition.

This form of the current can be used to show that fermions not only interact through their charge but also through their magnetic dipole moments. To show this, take the second term from Gordon decomposition, and show that its interaction with the field leads to:

$$j^{\mu}_{fi} A_{\mu} = \psi^{\dagger f}_{A} \left(\frac{e}{2m} \vec{\sigma} \cdot \vec{B} \right) \psi^{i}_{A}$$

where ψ_A is the upper piece of the Dirac spinor. *Hints: Treat this as a fermion elasticly scattering off the field. You will need the Dirac equation and its adjoint, and you will need the expressions for $g^{\mu\nu}$ and $\sigma^{\mu\nu}$ in terms of the γ^{μ} matrices.*