We will discuss some examples that involve equilibrium. We then move on to a discussion of friction.
Newton’s Laws of Motion

1) An object that is at rest will remain at rest, or an object that is moving will continue to move (in a straight line) with constant velocity, if and only if the net force acting on the object is zero.

2) An object of mass $m$ subjected to forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ will undergo an acceleration $\vec{a}$ given by

$$\vec{F}_{\text{net}} = m\vec{a} \quad \text{where} \quad \vec{F}_{\text{net}} = \sum_{i=1}^{n} \vec{F}_i$$

3) To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.
Let’s first consider problems of static equilibrium

- Equilibrium corresponds to the case that the object has zero net force acting on it $\mathbf{F}_{\text{net}} = 0$

- Two types of equilibrium conditions
  - Static equilibrium corresponds to the case when the object is at rest
  - Dynamic equilibrium corresponds to the case when the object moves at a constant velocity
  - Both correspond to $\mathbf{F}_{\text{net}} = 0$. 
Solving Problems

To solve problems using Newton’s laws of motion, we will follow the same problem solving procedure as for the case of kinematics.

1. Rewrite the problem in a couple of sentences;
   1. State the situation;
   2. What variables are we solving for;

2. Draw a force diagram Free Body Diagram

3. State the known and unknown variables;

4. Solve the problem algebraically;

5. Substitute numbers into the problem.
Example 1

Consider a lead block of mass 5 Kg held against a frictionless wall with a force applied at an angle of $30^\circ$ as shown in the figure. What is the magnitude of the force required to keep the block from sliding?
Brief description A 5 Kg block is held against a frictionless wall with a force acting at \(30^\circ\) relative to the horizontal. What is the magnitude of \(\vec{P}\) required to keep the block in static equilibrium?
Solution

Known
\( \theta = 30^\circ \)
\( \vec{w} = -mg\hat{j} = -49\hat{j} \text{ N} \)

Unknown
\( \vec{P}, \vec{n} \)

Equations:

\[
\begin{align*}
\sum F_x &= n - P_x = n - P \cos \theta = 0 \\
\sum F_y &= P_y - mg = P \sin \theta - mg = 0
\end{align*}
\]

\[
\Rightarrow \quad \left\{ \begin{array}{c}
n = P \cos \theta \\
P = mg / \sin \theta
\end{array} \right. \quad \Rightarrow \quad P = mg / \sin(30) = 98 \text{ N}
\]
Example 2

A 1000 Kg beam is supported by two ropes. Calculate the tension in each rope.
Example 3

A 1000 Kg beam is supported by two ropes. Calculate the tension in each rope.

Known
\( \alpha = 20^\circ \)
\( \beta = 30^\circ \)
\( \vec{w} = -mg \hat{j} = -9800 \hat{j} \text{ N} \)

Unknown
\( \vec{T}_1, \vec{T}_2 \)
**Example**

**Known**

\[ \alpha = 20^\circ \]
\[ \beta = 30^\circ \]
\[ \vec{w} = -mg \hat{j} = -9800 \hat{j} \text{ N} \]

**Unknown**

\( \vec{T}_1, \vec{T}_2 \)

\[
\begin{align*}
T_{2x} - T_{1x} &= T_1 \sin \alpha - T_2 \sin \beta = 0 \\
T_{1y} + T_{2y} - mg &= T_1 \cos \alpha + T_2 \cos \beta - mg = 0
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \\
T_1 &= 6396 \text{ N} \\
T_2 &= 4375 \text{ N}
\end{align*}
\]
Joe and Bill are playing tug-o-war. Joe is pulling with a force of 200 N. Bill is simply hanging on, but skidding towards Joe at a constant velocity. What is the magnitude of the force of friction between Bill’s feet and the ground.

1. 200 N
2. 400 N
3. 0 N
4. 300 N
5. None of the above
Next to gravity, friction is the most common force that we interact with. It allows us to walk, slows us down, and prevents us from moving.

There are 3 types of friction (all act to oppose motion):

- **Static friction** \( f_s \leq \mu_s N \) when no motion occurs;
- **Kinetic friction** \( f_k = \mu_k N \), when object is moving;
- **Rolling friction** \( f_r = \mu_r N \), when objects rolls.

### Table 5.1 Coefficients of friction

<table>
<thead>
<tr>
<th>Materials</th>
<th>Static</th>
<th>Kinetic</th>
<th>Rolling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber on concrete</td>
<td>1.00</td>
<td>0.80</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel on steel (dry)</td>
<td>0.80</td>
<td>0.60</td>
<td>0.002</td>
</tr>
<tr>
<td>Steel on steel (lubricated)</td>
<td>0.10</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.50</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Wood on snow</td>
<td>0.12</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Ice on ice</td>
<td>0.10</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>
For which situation is the frictional force the largest.
Solution

At rest, the object doesn’t move, so the static friction force increases to match the pushing force. This causes the graph to increase with a slope of μ \( g \).

\[ \text{Static friction} \]

The object slips when the friction force decreases as the object begins to move. The kinetic friction force remains constant once the object is moving.

\[ \text{Kinetic friction} \]

At rest, \( f = 0 \).

\[ f = \mu \cdot N \]

\[ \mu = \text{coefficient of friction} \]

\[ N = \text{normal force} \]

\[ \text{At rest} \]

\[ \text{Acceleraing} \]

\[ \text{Push} \]

\[ \text{Sliding} \]

\[ \text{Sliding} \]

\[ \text{Sliding} \]

\[ \text{Sliding} \]

\[ \text{Sliding} \]

\[ \text{Sliding} \]
Example 3

A 50 kg steel box is in the back of a dump truck. The truck’s bed, also made of steel, is slowly tilted. At what angle will the file cabinet begin to slide?

Brief description: A 50 kg steel box is on a steel incline plane. What is the maximum angle the incline can have for the box to remain in static equilibrium?
Solution

Known
\( \mu_s = 0.80, \ m = 50 \text{ kg} \)

Unknown
angle \( \theta \) of incline
Normal force \( n \)

Equations of motion:

\[
\begin{align*}
mg \sin \theta - \mu_s n &= 0 \\
-mg \cos \theta + n &= 0
\end{align*}
\]

\[ \Rightarrow \quad \mu_s = \tan \theta \quad \Rightarrow \quad \theta = \tan^{-1} \mu_s = 38.7^\circ \]
Read the sections on Newton’s second law and its applications:
Sections 5.2, 5.3, 5.6