Assignment 7

Due Friday October 30

1. Now use your code from Assignment 6 to solve for the motion of a particle in a $\frac{1}{r}$ potential. Assume the earth is a point mass and the acceleration is

$$a(r) = -G\frac{M}{r^2} \tag{1}$$

Start the particle at the surface of the earth and let the program go. Try shrinking the time-step to get better accuracy. Does the solution change? Try a smaller time-step. The solution is inaccurate because assuming a constant acceleration as the particle passes thru zero is wrong (what is the acceleration at $r = 0$?). As you make the time-step smaller, the distance of closest approach to zero will decrease and you'll need a smaller time-step, which is of course a *Catch-22*.

This problem is commonly treated in N-body problems by *softening* gravity where the potential is given by

$$\phi = -G\frac{M}{(r^2 + \epsilon^2)^{1/2}} \tag{2}$$

and the acceleration is given by

$$\vec{a}(\vec{r}) = -G\frac{M\vec{r}}{(r^2 + \epsilon^2)^{3/2}} \tag{3}$$

where ϵ is small compared to the smallest size scale you are trying to resolve in your simulation. Try this (with ϵ around $0.01R$).

This still won't solve the problem we had above because in both cases the impact parameter goes to zero, due to the initial conditions. So let's fix that up. Consider the problem in 2 dimensions and use Cartesian coordinates. To keep things simple we'll just follow circular orbits. Now start the particle with a velocity and position such that it has a circular orbit. Start at $R_\oplus$ and then make the radius smaller. What do you have to do to the timestep as the radius decreases? What happens if you try to make the radius smaller than epsilon?

What can you conclude about softening?