Takehome Version Midterm Exam #2
Due Wednesday Nov 11

I affirm under the penalties of Academic Misconduct as spelled out in the *Academic Misconduct Code of the University of Oklahoma* that: 1) I worked on this exam by myself; 2) that the only resources I used for completion of this assignment were my own notes taken in class and/or the Course Textbook by Francis LeBlanc. I affirm that I did not use the resources of the Internet for this exam.

Signature: ______________________________

Printed Name: ______________________________

1. (a) (10 points) For a polytropic equation of state, derive the relationship between pressure and energy density. *Hint:* $P = n^2 \left.\frac{\partial u/n}{\partial n}\right|_S$ and ignore the proportionality constant between n and ρ, $n = \frac{N_A}{\mu}\rho$, that is set $\frac{N_A}{\mu} = 1$.

   (b) (10 points) Multiply the equation of hydrostatic equilibrium by $4\pi r^3$ and derive the Virial Theorem.

   (c) (10 points) Use the Virial Theorem to find the total energy of a star with a polytropic equation of state in terms of the gravitational energy of the star and in terms of the internal energy of the star. Show that a polytrope with $\gamma = 4/3$ has a total energy of 0 and that a polytrope with $\gamma = 5/3$ corresponds to the classic case that the internal energy is half the gravitational energy in magnitude.

2. (a) (25 points) The entropy of a gas that consists of matter and radiation is given by

$$S = \text{constant} + \frac{N_A k}{\mu} \ln \frac{T^{3/2}}{\rho} + \frac{4a}{c}\frac{T^3}{\rho}$$

   where the last term is the entropy in radiation. The matter equataion of state is that of a standard ideal gas. Ignore the entropy in radiation and show that:

$$S = \frac{3}{2}\frac{N_A k}{\mu} \ln \frac{P}{\rho^{5/3}} + \text{constant}',$$

   that is, that $S = S(K)$. Explain the significance of the result $S = S(K)$ for the theory of polytropes.

   (b) (25 points) For a gas that consists of matter and radiation with gas fraction β, that is

$$P = P_g + P_{rad}$$

$$P_g = \beta P$$
$$P_{rad} = (1-\beta)P$$

Show that:

$$T = \left(\frac{N_A k}{\mu}\frac{3}{a}\frac{1-\beta}{\beta}\right)^{1/3}\rho^{1/3}$$

and that

$$P = \left[\left(\frac{N_A k}{\mu}\right)^4\frac{3}{a}\frac{1-\beta}{\beta^4}\right]^{1/3}\rho^{4/3}$$

and thus that $n=3$ corresponds to the Eddington standard model.