End of Convection Lecture, Oct 31

λ - total energy loss rate

$$\lambda = Sf$$

$$f = \frac{4acT^3}{3K\rho}\left|\frac{\partial T}{\partial n}\right|$$

n is normal to surface element

$$\frac{\partial T}{\partial n} \simeq \frac{2DT}{d}$$

$$\lambda = Sf = \frac{8acT^3}{3K\rho} DT \frac{S}{d}$$

$$\rho V c_p \frac{\partial T_e}{\partial t} = -\lambda$$ luminosity of blob

Temp decrease per unit length traversed
(blob moves w/ velocity v)

$$\frac{\lambda}{\rho V c_p v}$$

$$\left(\frac{dT}{dr}\right)_e = \left(\frac{dT}{dr}\right)_{ad} - \frac{\lambda}{\rho V c_p v}$$

multiply by

$\frac{H_p}{T}$ and get

$$\nabla_e - \nabla_{ad} = \frac{\lambda H_p}{\rho V c_p v T} \quad (5)$$

Now λ can be replaced
by the average DT given in (2)
and the form factor,

$$l_m \frac{S}{Vd} = \left(l_m/6\right)^{-1}$$ for a sphere of diameter l_m

In literature

$$\frac{l_m^5}{\nu d} \simeq \frac{9/2}{l_m}$$

Which we adopt

Then (5) becomes

$$\frac{\nabla_e - \nabla_{ad}}{\nabla - \nabla_e} = \frac{6acT^3}{K\rho^2 c_p l_m v}$$

$$l_m = \alpha H_p$$

$$1 \lesssim \alpha \lesssim 2$$

Summary

$$F_{rad} + F_{con} = \frac{4acG}{3} \frac{T^4 m}{\kappa P r^2} \nabla_{rad}$$

$$F_{rad} = \frac{4acG}{3} \frac{T^4 m}{\kappa P r^2} \nabla$$

$$v^2 = g\delta(\nabla - \nabla_e) \frac{l_m^2}{8 H_p}$$

$$F_{con} = \rho c_p T \sqrt{g\delta} \frac{l_m^2}{4\sqrt{2}} H_p^{-3/2} (\nabla - \nabla_e)^{3/2}$$

$$\frac{\nabla_e - \nabla_{ad}}{\nabla - \nabla_e} = \frac{6acT^3}{\kappa \rho^2 c_p l_m v}$$

Solve for

F_{rad}, F_{con}, v, ∇_e, ∇

given

P, T, ρ, ℓ, m, c_P, ∇_{ad}, ∇_{rad}, g