Wavelets, PCA, and SNe! Oh My!

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Wavelet Introduction

original

scale 1

scale 2

scale 3

scale 4

scale 5

scale 6

scale 7

scale 8

scale 9

residual

X(5150)

X(5150)

X(5485)

X(5750)

X(4570)
Wavelet Spectral Index Calculation

\[
X_{\{l\}} = \frac{\lambda_b}{\lambda_a} \sum \hat{W}_{\{l\}}(\lambda)
\]

Noise Corrections

X from spectra with Monte Carlo noise

Corrected Values
Wavelet Introduction

(at $B_{\text{max}}$)
Wavelet Introduction
(at $B_{\text{max}}$)

$X(5750)$

$X(6150)$

Legend:
- Prosale Cluster
- High Velocity
- Fast Decliners
- Slow Decliners
SNe Ia Subgroups
(at $B_{\text{max}}$)

Refined Subgroups (this work)

Branch et al. (2009)
### Spectral Index Correlations
(at $B_{\text{max}}$)

Number of SNe: 23  
Probability of 5% at: **0.961**

<table>
<thead>
<tr>
<th></th>
<th>$X(4570)$</th>
<th>$X(5150)$</th>
<th>$X(5485)$</th>
<th>$X(5750)$</th>
<th>$X(6150)$</th>
<th>$\Delta m_{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(4570)$</td>
<td>1</td>
<td>$-0.21991$</td>
<td>$0.40593$</td>
<td>$-0.49809$</td>
<td>0.07850</td>
<td>0.37612</td>
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<tr>
<td>$X(5150)$</td>
<td></td>
<td>1</td>
<td>$-0.51899$</td>
<td>0.64477</td>
<td>$-0.51569$</td>
<td>$-0.62048$</td>
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<tr>
<td>$X(5485)$</td>
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<td></td>
<td>1</td>
<td>$-0.43992$</td>
<td>0.33660</td>
<td>0.37926</td>
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<tr>
<td>$X(5750)$</td>
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<td></td>
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<td>1</td>
<td>$-0.68776$</td>
<td>$-0.92574$</td>
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<tr>
<td>$X(6150)$</td>
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<td></td>
<td></td>
<td>1</td>
<td>0.66223</td>
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<tr>
<td>$\Delta m_{15}$</td>
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<td>1</td>
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</tbody>
</table>
Principal Component Analysis (at $B_{max}$)

**Principal Component Coefficients**

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>PC1 coeff. =&gt;</td>
<td>0.25278</td>
<td>-0.42243</td>
<td>0.34222</td>
<td>-0.50426</td>
<td>0.39069</td>
<td>0.48334</td>
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<td>PC2 coeff. =&gt;</td>
<td>0.77164</td>
<td>0.09343</td>
<td>0.30993</td>
<td>0.00732</td>
<td>-0.53857</td>
<td>-0.09837</td>
</tr>
<tr>
<td>PC3 coeff. =&gt;</td>
<td>0.29331</td>
<td>0.34939</td>
<td>-0.74737</td>
<td>-0.32664</td>
<td>-0.01858</td>
<td>0.35537</td>
</tr>
<tr>
<td>PC4 coeff. =&gt;</td>
<td>-0.04373</td>
<td>-0.80403</td>
<td>-0.40077</td>
<td>0.05257</td>
<td>-0.43377</td>
<td>0.00939</td>
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<tr>
<td>PC5 coeff. =&gt;</td>
<td>-0.43441</td>
<td>0.20133</td>
<td>0.25469</td>
<td>-0.04235</td>
<td>-0.55624</td>
<td>0.62824</td>
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<tr>
<td>PC6 coeff. =&gt;</td>
<td>0.25302</td>
<td>-0.06124</td>
<td>-0.05269</td>
<td>0.79650</td>
<td>0.24373</td>
<td>0.48542</td>
</tr>
</tbody>
</table>

**Percentage of Variation**

*PCA Eigenvalues*

| PC1     | 3.5085984 | = 58.48% |
| PC2     | 1.1385310 | = 18.98% |
| PC3     | 0.71907840 | = 11.98% |
| PC4     | 0.41622111 | = 6.94% |
| PC5     | 0.17090850 | = 2.85% |
| PC6     | 0.046662567 | = 0.78% |
Spectral Index Evolution
X(4570) & X(5150) Evolution

[Graph showing the evolution of X(4570) and X(5150) over epochs in days, with data points for various years such as 1989B, 1990N, 1996X, and so on.]
Functional form for evolution.

\[ X(t) = \alpha \left( \frac{1}{\beta - t} - 1 \right) \left( 1 + e^{\gamma} \right) \]
\[ X(t) = \frac{\alpha}{\beta - t} + \epsilon \left(1 + e^{\gamma t}\right) \]
\[ X(t) = \frac{\alpha}{\beta - t} + \epsilon \]

\[ 1 + e^{\gamma} \]
Spectral Index Correlations
(evolution of $\Delta m_{15}$ correlations)

- Critical correlations for all spectra ($\leq 5\%$)
- Critical correlations w/o HV spectra ($\leq 5\%$)

epochs of strong correlation
epochs of transition
epochs of strong correlation
Principal Componants Analysis
(evolution of PC1)

S II 'w' is lost in later epochs
Principal Componants Analysis
(evolution of PC2)

The rest of the PC plots are less interesting than this.
Principal Component Analysis
(PC1 fits)

X(4570)

X(5150)
Principal Componants Analysis
(PC1 fits)

X(5485)

X(5750)
Predicting $\Delta m_{15}$