The Minimal Supersymmetric Standard Model (MSSM)

Construction

★ gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\begin{aligned} B_\mu &\rightarrow \hat{B} \ni (\lambda_0, B_\mu, \mathcal{D}_B), \\ W_{A\mu} &\rightarrow \hat{W}_A \ni (\lambda_A, W_{A\mu}, \mathcal{D}_{WA}),\ A = 1,2,3,\ \text{and} \\ g_{A\mu} &\rightarrow \hat{g}_A \ni (\tilde{g}_A, G_{A\mu}, \mathcal{D}_{gA}),\ A = 1,\cdots,8. \end{aligned}$$

★ matter content: 3 generations quarks and leptons

$$\begin{aligned} \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} &\rightarrow \hat{L}_i \equiv \begin{pmatrix} \hat{\nu}_i \\ \hat{e}_i \end{pmatrix}, \\ (e_R)^c &\rightarrow \hat{E}^c_i, \\ \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} &\rightarrow \hat{Q}_i \equiv \begin{pmatrix} \hat{u}_i \\ \hat{d}_i \end{pmatrix}, \end{aligned}$$

$$
\begin{aligned}
(u_R)^c &\rightarrow \hat{U}_i^c, \\
(d_R)^c &\rightarrow \hat{D}_i^c,
\end{aligned}
$$

where $e.g.$

$$\hat{e} = \tilde{e}_L(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_{eL}(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}_e(\hat{x}) \tag{1}$$

while

$$\hat{E}^c = \tilde{e}_R^\dagger(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_{E^cL}(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}_{E^c}(\hat{x}). \tag{2}$$

SM Dirac fermions are constructed out of Majorana fermions via

$$e = P_L\psi_e + P_R\psi_{E^c}. \tag{3}$$

where in chiral rep. of γ matrices

$$\psi_e = \begin{pmatrix} e_1 \\ e_2 \\ -e_2^* \\ e_1^* \end{pmatrix} \text{ and } \quad \psi_{E^c} = \begin{pmatrix} e_4^* \\ -e_3^* \\ e_3 \\ e_4 \end{pmatrix}.$$

The MSSM (part 2)

Construction

⋆ Higgs multiplets:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix} . \tag{4}$$

Now spin $\frac{1}{2}$ higgsinos with $Y = 1$ can circulate in triangle anomalies; cancel with additional $Y = -1$ doublet:

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix} , \tag{5}$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{L} = \begin{pmatrix} \hat{\nu}_{eL} \\ \hat{e}_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	-1
$\hat{E}^c$	$\mathbf{1}$	$\mathbf{1}$	2
$\hat{Q} = \begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$
$\hat{U}^c$	$\mathbf{3}^*$	$\mathbf{1}$	$-\frac{4}{3}$
$\hat{D}^c$	$\mathbf{3}^*$	$\mathbf{1}$	$\frac{2}{3}$
$\hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	1
$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}^*$	**-1**

The MSSM (part 3)

Construction

★ superpotential

$$\hat{f} = \mu \hat{H}_u^a \hat{H}_{da} + \sum_{i,j=1,3} \left[(\mathbf{f}_u)_{ij} \epsilon_{ab} \hat{Q}_i^a \hat{H}_u^b \hat{U}_j^c + (\mathbf{f}_d)_{ij} \hat{Q}_i^a \hat{H}_{da} \hat{D}_j^c + (\mathbf{f}_e)_{ij} \hat{L}_i^a \hat{H}_{da} \hat{E}_j^c \right]. \tag{6}$$

The following terms are gauge invariant and renormalizable, but violate baryon and lepton number. They are excluded if one requires R-parity conservation $R = (-1)^{3(B-L)+2s}$:

$$\hat{f}_{\not{L}} = \sum_{i,j,k} \left[\lambda_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \lambda'_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{Q}_j^b \hat{D}_k^c \right] + \sum_i \mu'_i \epsilon_{ab} \hat{L}_i^a \hat{H}_u^b, \tag{7}$$

and

$$\hat{f}_{\not{B}} = \sum_{i,j,k} \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c, \tag{8}$$

The MSSM (part 4)

⋆ soft SUSY breaking terms

$$
\begin{aligned}
\mathcal{L}_{\rm soft} &= -\Big[\tilde{Q}_i^\dagger {\mathbf{m^2_Q}}_{ij}\tilde{Q}_j + \tilde{d}_{Ri}^\dagger {\mathbf{m^2_D}}_{ij}\tilde{d}_{Rj} + \tilde{u}_{Ri}^\dagger {\mathbf{m^2_U}}_{ij}\tilde{u}_{Rj} \\
&+ \tilde{L}_i^\dagger {\mathbf{m^2_L}}_{ij}\tilde{L}_j + \tilde{e}_{Ri}^\dagger {\mathbf{m^2_E}}_{ij}\tilde{e}_{Rj} + m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2\Big] \\
&- \frac{1}{2}\left[M_1\bar{\lambda}_0\lambda_0 + M_2\bar{\lambda}_A\lambda_A + M_3\bar{\tilde{g}}_B\tilde{g}_B\right] \\
&- \frac{i}{2}\left[M_1'\bar{\lambda}_0\gamma_5\lambda_0 + M_2'\bar{\lambda}_A\gamma_5\lambda_A + M_3'\bar{\tilde{g}}_B\gamma_5\tilde{g}_B\right] \\
&+ \Big[(\mathbf{a_u})_{ij}\epsilon_{ab}\tilde{Q}_i^a H_u^b \tilde{u}_{Rj}^\dagger + (\mathbf{a_d})_{ij}\tilde{Q}_i^a H_{da}\tilde{d}_{Rj}^\dagger + (\mathbf{a_e})_{ij}\tilde{L}_i^a H_{da}\tilde{e}_{Rj}^\dagger + \text{h.c.}\Big] \\
&+ \left[bH_u^a H_{da} + \text{h.c.}\right],
\end{aligned}
$$

The MSSM (part 5)

- count parameters
  - g_1, g_2, g_3, θ_{QCD}
  - gaugino masses M_1, M_1', M_2, M_2', M_3 (M_3' absorbed into $\tilde{g}$)
  - $m^2_{H_u}$, $m^2_{H_d}$, μ, b (phase of b absorbed)
  - $5 \times (6+3) = 45$ in sfermion mass matrices
  - $3 \times (3 \times 3 \times 2) = 54$ in Yukawa matrices
  - $3 \times (3 \times 3 \times 2) = 54$ in a-term matrices
  - a global $U(3)^5$ transformation in matter allows $45 - 2 = 43$ phases absorbed into matter sfermions
  - total parameters $= 9 + 5 + 45 + 54 + 54 - 43 = 124$
- most choices are excluded: lead to FCNC or CP violating effects
  - solutions: universality, decoupling, alignment

The MSSM (part 6): electroweak breaking

- ⋆ construct scalar potential of MSSM: $V = V_F + V_D + V_{soft}$
- ⋆ minimization conditions: $\partial V/\partial h_u^0 = \partial V \partial h_d^0 = 0$ has solution so $\langle h_u^0 \rangle = v_u$, $\langle h_d^0 \rangle = v_d$ with $\tan\beta \equiv v_u/v_d$
  - $W^\pm$, Z_0 become massive as in SM
  - SM fermions all gain mass *e.g.* $m_e = f_e v_d$
- ⋆ states with same spin/charge can mix
  - predict many new states to exist!

The MSSM (part 7): new matter states

- ⋆ spin $\frac{1}{2}$ massive color octet: gluino $\tilde{g}$
- ⋆ spin $\frac{1}{2}$ bino, wino, neutral higgsinos $\Rightarrow$ neutralinos $\widetilde{Z}_1,\ \widetilde{Z}_2,\ \widetilde{Z}_3,\ \widetilde{Z}_4$
- ⋆ spin $\frac{1}{2}$ charged wino, higgsinos $\Rightarrow$ charginos $\widetilde{W}_1^\pm,\ \widetilde{W}_2^\pm$
- ⋆ spin-0 squarks: $\tilde{u}_L,\ \tilde{u}_R,\ \tilde{d}_L,\ \tilde{d}_R, \tilde{s}_L,\ \tilde{s}_R,\ \tilde{c}_L,\ \tilde{c}_R,\ \tilde{b}_1,\ \tilde{b}_2,\ \tilde{t}_1,\ \tilde{t}_2$
- ⋆ spin-0 sleptons: $\tilde{e}_L,\ \tilde{e}_R,\ \tilde{\nu}_e,\ \tilde{\mu}_L,\ \tilde{\mu}_R,\ \tilde{\nu}_\mu,\ \tilde{\tau}_1,\ \tilde{\tau}_2, \tilde{\nu}_\tau$
- ⋆ spin-0 higgs bosons: $h,\ H,\ A,\ H^\pm$ (h usually SM-like)

The MSSM: summary

- ⋆ The MSSM includes the SM as a sub-theory, but also includes many new states of matter
- ⋆ Unlike the SM, the MSSM is free of quadratic divergences in the scalar sector
- ⋆ Thus, the MSSM can accommodate vastly different mass scales, *e.g.* M_{weak} and M_{GUT} or M_{string}
- ⋆ The 124 parameter MSSM is likely to be the low energy effective theory of some more fundamental theory, perhaps one linked to GUTs or strings
- ⋆ The MSSM provides for us the possible physical states and Feynman rules needed for making predictions of physical phenomena
- ⋆ The MSSM parameters are highly constrained by bounds from FCNCs, CP-violation, etc.

The MSSM: RGEs

- ⋆ If the MSSM is to be valid between vastly different mass scales, then it is important to relate parameters between these scales.
- ⋆ The gauge couplings, Yukawa couplings, μ term and soft breaking parameter evolution is governed by *renormalization group equations*, or RGEs
- ⋆ For gauge couplings, these have the form

$$\frac{dg_i}{dt} = \beta(g_i) \quad with \quad t = \log Q \tag{9}$$

- ⋆ In SM,

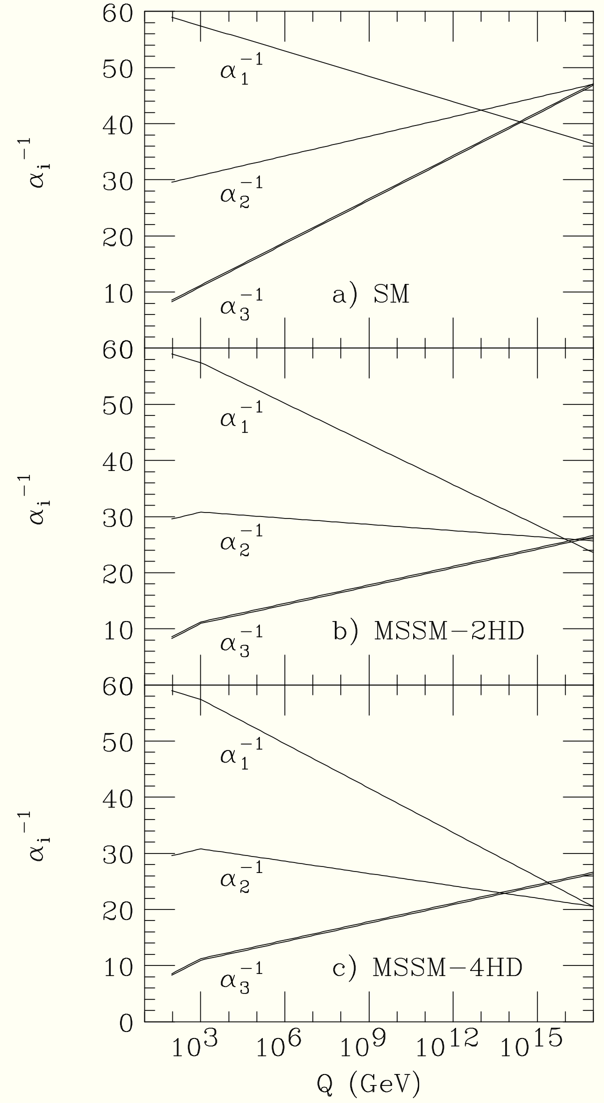
$$\beta(g) = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}C(G) - \frac{2}{3}n_F S(R_F) - \frac{1}{3}n_H S(R_H)\right]. \tag{10}$$

- ⋆ In MSSM, the gauginos, matter and Higgs scalars also contribute:

$$\beta(g) = -\frac{g^3}{16\pi^2}\left[3C(G) - S(R)\right], \tag{11}$$

- ⋆ Can use the precision values of g_1, g_2 and g_3 measured at $Q = M_Z$ at LEP2 as boundary conditions, and extrapolate to high energy

Gauge coupling evolution



The MSSM: RGEs continued

$$
\begin{aligned}
\frac{dM_i}{dt} &= \frac{2}{16\pi^2} b_i g_i^2 M_i, \\
\frac{dA_t}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i g_i^2 M_i + 6f_t^2 A_t + f_b^2 A_b\right), \\
\frac{dA_b}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i' g_i^2 M_i + 6f_b^2 A_b + f_t^2 A_t + f_\tau^2 A_\tau\right), \\
\frac{dA_\tau}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i'' g_i^2 M_i + 3f_b^2 A_b + 4f_\tau^2 A_\tau\right), \\
\frac{dB}{dt} &= \frac{2}{16\pi^2}\left(-\frac{3}{5} g_1^2 M_1 - 3g_2^2 M_2 + 3f_b^2 A_b + 3f_t^2 A_t + f_\tau^2 A_\tau\right), \\
\frac{d\mu}{dt} &= \frac{\mu}{16\pi^2}\left(-\frac{3}{5} g_1^2 - 3g_2^2 + 3f_t^2 + 3f_b^2 + f_\tau^2\right),
\end{aligned}
$$

$$\frac{dm_{Q_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{1}{15}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{10}g_1^2S + f_t^2X_t + f_b^2X_b\right),$$

$$\frac{dm_{\tilde{t}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{16}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 - \frac{2}{5}g_1^2S + 2f_t^2X_t\right),$$

$$\frac{dm_{\tilde{b}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{4}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{5}g_1^2S + 2f_b^2X_b\right),$$

$$\frac{dm_{L_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S + f_\tau^2X_\tau\right),$$

$$\frac{dm_{\tilde{\tau}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{12}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2S + 2f_\tau^2X_\tau\right),$$

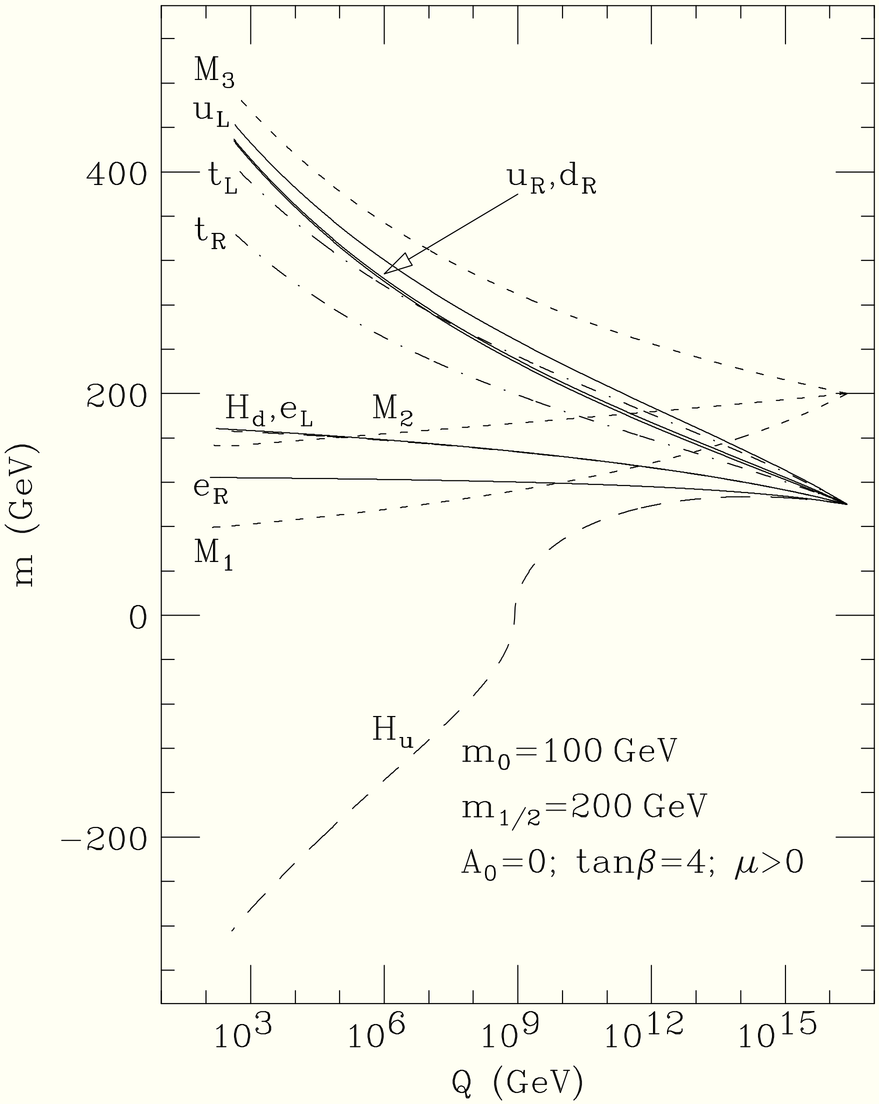
$$\frac{dm_{H_d}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S + 3f_b^2X_b + f_\tau^2X_\tau\right),$$

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right),$$

where m_{Q_3} and m_{L_3} denote the mass term for the third generation $SU(2)$ squark and slepton doublet respectively, and

$$
\begin{aligned}
X_t &= m_{Q_3}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2 + A_t^2, \\
X_b &= m_{Q_3}^2 + m_{\tilde{b}_R}^2 + m_{H_d}^2 + A_b^2, \\
X_\tau &= m_{L_3}^2 + m_{\tilde{\tau}_R}^2 + m_{H_d}^2 + A_\tau^2, \text{ and} \\
S &= m_{H_u}^2 - m_{H_d}^2 + Tr\left[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2\right].
\end{aligned}
$$

Soft term evolution and radiative EWSB for $m_t \sim 175$ GeV



Supergravity

- ⋆ In SUSY transformation operator $e^{-i\bar{\alpha}Q}$ let $\alpha = \alpha(x)$ so we have a *local* SUSY transformation
- ⋆ Just as for gauge theories, will need to introduce a gauge field to maintain covariance: $\psi_\mu(x)$, a spin $\frac{3}{2}$ vector-spinor (Rarita-Schwinger) field
- ⋆ To maintain local SUSY, will have to introduce bosonic partner: a spin 2 field $g_{\mu\nu}(x)$
  - $g_{\mu\nu}$ is massless, and in classical limit obeys Einstein GR eq'ns of motion: it is the graviton field
  - usually, $g_{\mu\nu}(x)$ is traded for the equivalent vierbein field $e_\mu^a(x)$, where $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$, where η_{ab} is the Minkowski metric
- ⋆ Can derive a Master formula for *supergravity* (SUGRA) gauge theories

Supergravity

- SUGRA is inherently non-renormalizable
- SUGRA theories specified by Kähler function

$$G(\hat{\mathcal{S}}^\dagger, \hat{\mathcal{S}}) = K(\hat{\mathcal{S}}^\dagger, \hat{\mathcal{S}}) + \log|\hat{f}(\hat{\mathcal{S}})|^2, \tag{12}$$

  and gauge kinetic function

$$f_{AB}(\hat{S}). \tag{13}$$

- SUGRA can be spontaneously broken just as SUSY can
- Since SUGRA is local SUSY theory, have a super-Higgs mechanism, wherein the gravitino field ψ_μ gains a mass $m_{3/2}$ while graviton remains massless
- Can embed MSSM in a SUGRA theory along with gauge singlet field(s) $\hat{h}_m$ with superpotential such that SUGRA is spontaneously broken (hidden sector)
- SUGRA breaking communicated from hidden sector to visible sector via gravity: induces soft SUSY breaking terms of order $\sim m_{3/2}$!

Minimal Supergravity model (mSUGRA)

- Assume MSSM embedded in a SUGRA theory
- SUSY broken in hidden sector with $m_{3/2} \sim M_{weak} \sim 1$ TeV
- For simple choice of Kähler function and gauge kinetic function, will induce $universal$ scalar masses m_0, gaugino masses $m_{1/2}$ and trilinears A_0
- Inspired by gauge coupling unification, these universal choices usually taken at $Q = M_{GUT} \simeq 2 \times 10^{16}$ GeV
- Evolve couplings and soft parameters from M_{GUT} to M_{weak}; $m_{H_u}^2 \to$ negative, breaking EW symmetry.
- All sparticle masses, mixings at $Q = M_{weak}$ calculated in terms of small parameter set:

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ sign(\mu) \tag{14}$$

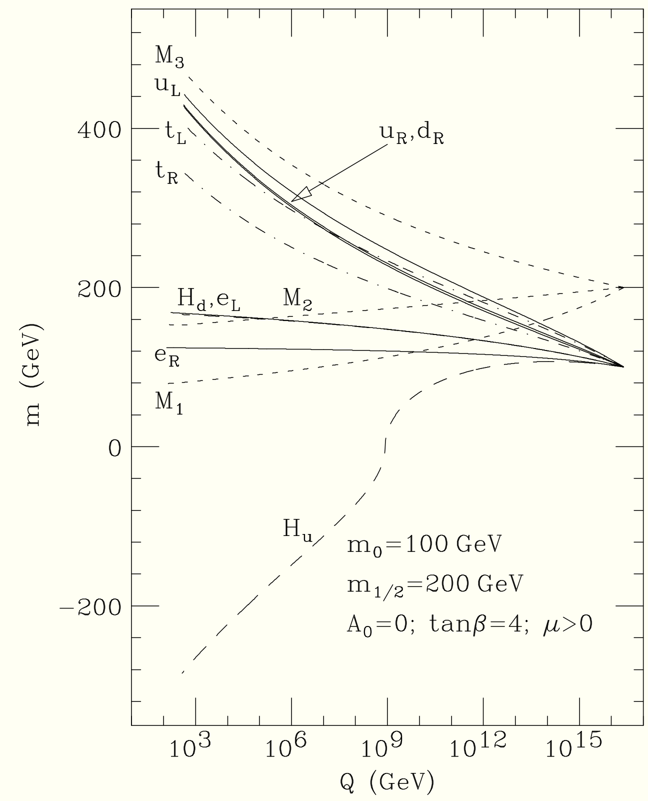
- The mSUGRA model is paradigm SUSY model for phenomenological analysis, but is not likely to be the complete story.

Sparticle mass spectra

★ Mass spectra codes

★ RGE running: $M_{GUT} \to M_{weak}$

- Isajet (HB, Paige, Protopopescu, Tata)
  - $*$ $\geq$7.72: Isatools
- SuSpect (Djouadi, Kneur, Moultaka)
- SoftSUSY (Allanach)
- Spheno (Porod)



★ Comparison (Belanger, Kraml, Pukhov)

★ Website: `http://kraml.home.cern.ch/kraml/comparison/`

$SU(5)$ SUSY grand unified theory

★ $SU(5)$: smallest simple gauge group that contains $SU(3) \times SU(2) \times U(1)$

– Georgi-Glashow, 1975

★ Unifies 3 SM gauge couplings into just one

★ Explains seemingly ad-hoc SM/MSSM weak hypercharge assignments

- Some common irreps: $\psi^i = \mathbf{5}$, $\psi_i = \bar{\mathbf{5}}$, $A^{ij} = \mathbf{10}$, $S^{ij} = \mathbf{15}$, $\psi^i_j = \mathbf{24}$
- $\hat{W}_i,\ i = 1 - 24$ gauge superfields
- $\hat{D}^c,\ \hat{L} \in \bar{\mathbf{5}}$
- $\hat{Q},\ \hat{U}^c,\ \hat{E}^c \in \mathbf{10}$
- $H_d \in \hat{H}_1 \equiv \bar{\mathbf{5}}$
- $H_u \in \hat{H}_2 \equiv \mathbf{5}$
- $\hat{\Sigma}(24)$ needed to break $SU(5)$

$SU(5)$ continued

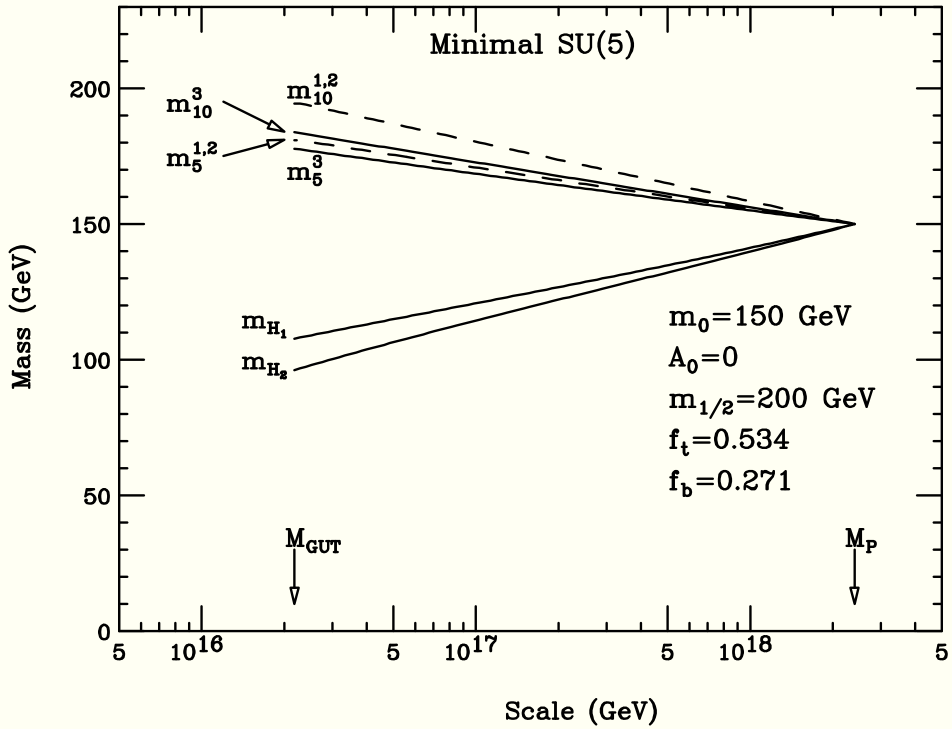
★ superpotential

$$\begin{aligned}\hat{f} &= \mu_\Sigma tr\hat{\Sigma}^2 + \frac{1}{6}\lambda_\Sigma tr\hat{\Sigma}^3 + \mu_H \hat{\mathcal{H}}_1\hat{\mathcal{H}}_2 + \lambda\hat{\mathcal{H}}_1\hat{\Sigma}\hat{\mathcal{H}}_2 \\ &+ \frac{1}{4}f_t\epsilon_{ijklm}\hat{\psi}^{ij}\hat{\psi}^{kl}\hat{\mathcal{H}}_2^m + \sqrt{2}f_b\hat{\psi}^{ij}\hat{\phi}_i\hat{\mathcal{H}}_{1j} + \cdots,\end{aligned}$$

★ soft terms

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -m_{\mathcal{H}_1}^2|\mathcal{H}_1|^2 - m_{\mathcal{H}_2}^2|\mathcal{H}_2|^2 - m_\Sigma^2 tr\{\Sigma^\dagger\Sigma\} - m_5^2|\phi|^2 - m_{10}^2 tr\{\psi^\dagger\psi\} \\ &- \frac{1}{2}M_5\bar{\lambda}_\alpha\lambda_\alpha \\ &+ \Big[B_\Sigma\mu_\Sigma tr\Sigma^2 + \frac{1}{6}A_{\lambda_\Sigma}\lambda_\Sigma tr\Sigma^3 + B_H\mu_H\mathcal{H}_1\mathcal{H}_2 + A_\lambda\lambda\mathcal{H}_1\Sigma\mathcal{H}_2 \\ &+ \frac{1}{4}A_t f_t\epsilon_{ijklm}\psi^{ij}\psi^{kl}\mathcal{H}_2^m + \sqrt{2}A_b f_b\psi^{ij}\phi_i\mathcal{H}_{1j} + \text{h.c.}\Big].\end{aligned}$$

Sample soft term evolution



★ Boundary conditions:

$$
\begin{aligned}
m_{10} &= m_5 = m_{\mathcal{H}_1} = m_{\mathcal{H}_2} = m_\Sigma \equiv m_0 \\
A_t &= A_b = A_\lambda = A_{\lambda_\Sigma} \equiv A_0,
\end{aligned}
$$

$SU(5)$: problems

- ⋆ extra gauge bosons can induce p-decay: analyses rule out 4-d $SU(5)$?
- ⋆ Color triplet Higgs in H_1, H_2 must decouple, but they are in same multiplet as doublets (doublet/triplet splitting problem)
- ⋆ Synopsis: many believe some aspect of $SU(5)$ must be right. Conventionally, (SUSY)GUTs have been formulated in 4-dimensions, where large Higgs reps are needed for GUT symmetry breaking, and can lead to problems. In past 5 years, SUSY GUTs have been formulated in 5 or more dimensions. The compactification of the extra dimensions on appropriate orbifolds can be used instead to break the GUT symmetry, dispense with large higgs reps, and maintain proton stability. It is possible the GUT symmetry holds in extra dimensions as a remnant of string theory, but is broken by further compactifications. Recent work by Hall$+$ Nomura, Altarelli$+$Feruglio, March-Russell$+$Hebecker, Kawamura, show that extra dim'l GUTs can solve many problems associated with 4-d GUTs.

$SO(10)$: synopsis

⋆ $SO(10)$ is a rank-5 Lie group which contains the SM gauge symmetry. It has several important features:

- The $SO(2n)$ groups have *spinorial* representations of dim'n 2^{n-1}, in addition to the usual tensor reps of $SU(n)$.
- The 16-dim'l spinor rep of $SO(10)$ is large enough to contain *all* the matter in a single generation of the SM, plus a right-handed neutrino state. This *unifies* matter as well as gauge groups.
- The right-hand neutrino state is contained in a superfield

$$\hat{N}_i^c = \tilde{\nu}_{Ri}^\dagger(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_{N_i^c L}(\hat{x}) + i\bar{\theta}\theta_L \mathcal{F}_{N_i^c}(\hat{x}).$$

- Upon breaking $SO(10)$, the $\hat{N}_i^c$ fields become SM singlets, and can obtain a Majorana mass M_{Ni}. The superpotential obtains the form

$$\hat{f} = \hat{f}_{\mathrm{MSSM}} + (\mathbf{f}_\nu)_{ij}\epsilon_{ab}\hat{L}_i^a\hat{H}_u^b\hat{N}_j^c + \frac{1}{2}M_{Ni}\hat{N}_i^c\hat{N}_i^c\ . \tag{15}$$

$SO(10)$: continued

- Upon EWSB, the neutrinos obtain masses via the *see-saw* mechanism, where the (dominantly) right-handed neutrino obtains a mass $m_{\nu R} \sim M_N$, while the (dominantly) left-handed neutrino obtains a mass $m_{\nu L} \sim \frac{(f_\nu v_u)^2}{M_{Ni}}$. For third generation, with $f_\nu \simeq f_t$, then $m_{\nu_\tau} \sim 0.03$ eV for $M_{N3} \sim 10^{15}$ GeV, very close to M_{GUT}!
- Further, the group $SO(n)$ (except $n = 6$) are naturally anomaly-free, thus explaining the seemingly fortuitous anomaly cancellation in the SM and in $SU(5)$.
- In the unbroken $SO(10)$ theory, the superpotential is expected to have the form

$$\hat{f} \ni f\hat{\psi}_{16}\hat{\psi}_{16}\hat{\phi}_{10} + \ldots \tag{16}$$

with f being the single Yukawa coupling per generation in the GUT scale theory. The ellipses represent terms including for instance higher dimensional Higgs representations and interactions responsible for the breaking of

$SO(10)$. Thus, naively, it is expected in $SO(10)$ theories that the various Yukawa couplings of each generation should unify as well. This should hold especially for the 3rd generation. Yukawa coupling unification puts a strong constraint on the phenomenology expected in SUSY models. See $e.g.$ A. Auto, HB, C. Balazs, A. Belyaev, J. Ferrandis and X. Tata, JHEP**0306**, 023 (2003), hep-ph/0302155.