Impossibility of a biased Stark trap in two dimensions

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We show that it is impossible to create a two-dimensional static electric field $\vec{E}$ with a local minimum in E^2 at a point in space for which $\vec{E} \neq 0$. This fact may have consequences in the design of state selectors and guides of slow-moving molecules.

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The production of slow-moving ground-state polar molecules can be achieved by several means, including photoassociation, decceleration of a molecular beam using a traveling potential well, and He buffer gas cooling [1]. A long straight electrostatic guide (previously used as a molecular beam state selector [2]) has many applications to ultracold molecular physics, including the transport of molecules from one vacuum chamber to another, high-resolution spectroscopy of guided molecules, and ultracold collisions. The spatial dependence of a static electric field $\vec{E}$ leads to a local minimum in the Stark energy at the center of a long straight guide. A population of slow moving molecules in low field seeking states (i.e., in states for which the Stark potential is a monotonically increasing function of E^2) is confined by such a minimum, thus preventing divergence of the molecules.

One possible complication of a Stark guide is nonadiabatic (Majorana) transitions that occur in regions of space where low field seeking trapped states become degenerate with high field seeking untrapped states [3,4]. These nonadiabatic transitions are especially likely to occur in regions of zero field. One might imagine overcoming this problem by creating a minimum in E^2 at a point that the electric field is not zero. Trapped molecules would then be confined to regions of space for which the field is never zero. In addition to suppressing nonadiabatic transitions, such a biased guide might also provide an environment for studying collisions and spectroscopy of field-aligned particles.

Designs of electrodes that create a three-dimensional local minimum in E^2 at a point at which $\vec{E} \neq 0$ have already been reported [5,6]. For the present case of a long straight guide extending in the $\hat{z}$ direction, $E_z=0$ and $\vec{E}$ depends only on x and y. In what follows we prove that this two-dimensional case does not allow for a local minimum in E^2 at a point at which $\vec{E} \neq 0$.

Solutions to Laplace's equation in two dimensions that do not diverge at $\vec{r}=0$ and have $\vec{E}=E_o\hat{x}$ at the origin can be written as

$$V = \sum_{n=1}^{\infty} A_n r^n \cos(n\phi + \delta_n), \tag{1}$$

where $A_1=-E_o$, $\delta_1=0$, and $\vec{r}$ is given by the polar coordinates ϕ and r. We consider the possibility of a local minimum in E^2. The expansion of V leads to

$$E^2 = \frac{1}{2}\nabla^2 V^2 \tag{2}$$

$$= E_o^2 + \sum_{m=1}^{\infty} C_m r^m, \tag{3}$$

where

$$C_m = \sum_{i=1}^{m+1} i(m+2-i) A_i A_{m+2-i} \times \cos\{[2(i-1)-m]\phi + \delta_i - \delta_{m+2-i}\}. \tag{4}$$

For the potential energy to have a minimum at $r=0$, the coefficient of lowest nonzero power of r in Eq. (3) must be positive definite. Assume $A_2 \neq 0$. Then the lowest order term in Eq. (3) is r^1, and $C_1=-4E_oA_2\cos(\phi+\delta_2)$. Because this term can not be a positive definite function of ϕ, $A_2=0$. Now assume that $A_2=A_3=\cdots=A_k=0$. Then the lowest order term must be r^k with coefficient $C_k=-2(k+1)E_oA_{k+1}\cos(k\phi+\delta_{k+1})$. Because this term can not be a positive definite function of ϕ, $A_{k+1}=0$. The impossibility of a local minimum in E^2 with $\vec{E} \neq 0$ is proven by induction.

Although we have proven that it is impossible to construct a two-dimensional guide with a nonzero electric field at the point that E^2 is a minimum, we have not considered gravity or particles moving in a curved guide [7]. Such considerations may lead to ways to prevent molecules from crossing regions of zero electric field.

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