Dimensional Analysis

Dimensional Analysis is the process of using the dimensions of a quantity to

1) Check your answer

or

2) Determine an equation

1) Suppose you were solving for speed, and you got an answer of something like 4.0 m^2/s. You would know, just by looking at the units, that your calculation was wrong since speed must have units of m/s.

Suppose you want to calculate how far a car will travel that is going a certain speed for a certain time, but you can't remember if the equations for calculating this is $x = (\frac{1}{2})vt^2$ or $x = (\frac{1}{2})vt$.

Write the dimensions of these equations with dimensional quantities in square brackets using [L] for length and [T] for time.

Dimensions for $x = (\frac{1}{2})vt^2$: $[L] = \{[L]/[T]\} \times [T]^2 = [LT]$

Dimensions for $x = (\frac{1}{2})vt$: $[L] = \{[L]/[T]\} \times [T] = [L]$

so the correct equation must *not* have the t^2 term. Dimensional analysis doesn't tell us which constant terms to use, so $x = vt$, (without the ½ is also valid).

Interactive Question

If x is a distance, v is a velocity, and t is a time, which of the following could possibly be a correct equation according to dimensional analysis?

A) $x = vt^2$
B) $x = v^2t$
C) $x = vt$
D) $x = vt/2$
E) either C or D

Interactive Question Answer

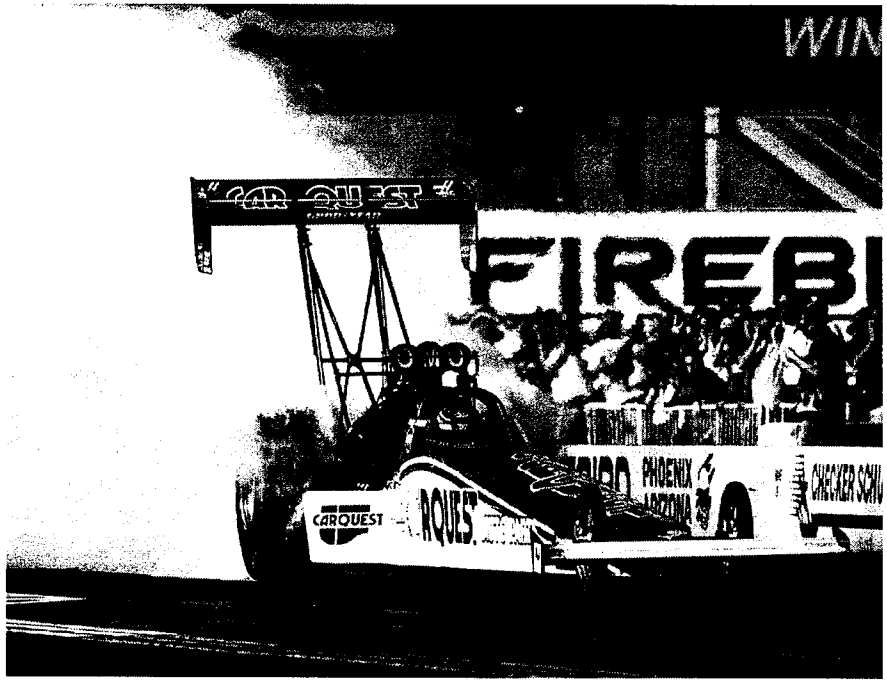
If x is a distance, v is a velocity, and t is a time, which of the following could possibly be a correct equation according to dimensional analysis?

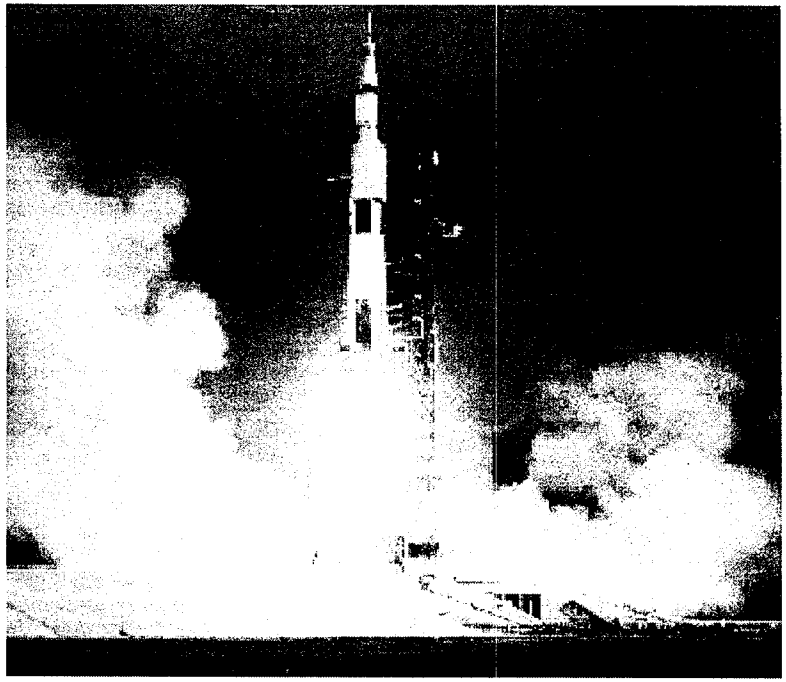
A) $x = vt^2$	$[L] = \{[L]/[T]\}[T]^2 = [L][T]$
B) $x = v^2t$	$[L] = \{[L]/[T]\}^2[T] = [L]^2/[T]$
C) $x = vt$	$[L] = \{[L]/[T]\}[T] = [L]$
D) $x = vt/2$	$[L] = \{[L]/[T]\}[T] = [L]$

E) either C or D

Chapter 2

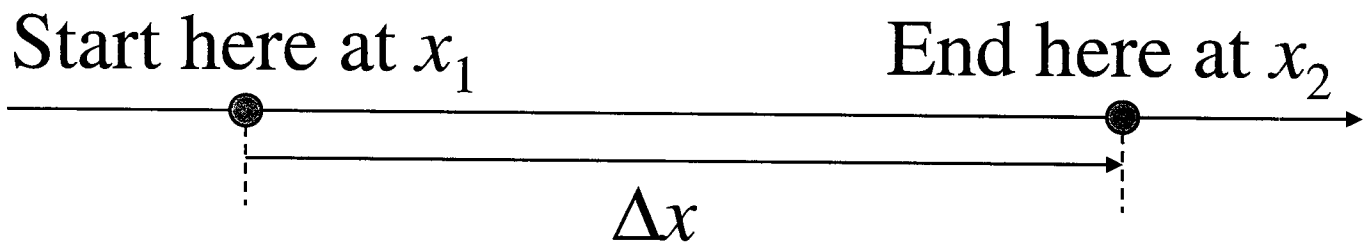
Motion in One dimension





Distance and Displacement

- Displacement is the net change in position: $\Delta x = x_2 - x_1$
- x_2 is the position at t_2 and x_1 is the position at t_1 with t_2 occurring after t_1.



- Displacement can have a positive or negative sign.
- Displacement is not the same as total distance traveled (Δs).

If you jog a lap around a track, your displacement is zero, but the distance you have traveled is not zero. We will mostly use displacement in this class.

Interactive Question

An object goes from one point in space to another. After it arrives at its destination, its *displacement* is:

A) either greater than or equal to
B) always greater than
C) always equal to
D) either smaller than or equal to
E) always smaller than

the *distance* it traveled.

Interactive Question Answer

An object goes from one point in space to another. After it arrives at its destination, its *displacement* is:

D) either smaller than or equal to

the *distance* it traveled.

Average Velocity

The average velocity is the displacement divided by the time it takes to go from a beginning position to a final position.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Velocity is a vector and has a magnitude and direction. In one dimension, the only "direction" is the positive or negative direction. But in more than one dimension, the direction is more complicated. This direction doesn't really affect the problem in one dimension, but will be very important when we work in two or more dimensions.

Average speed

The average speed is the distance (Δs) divided by the time it takes to travel that distance (Δt) .

$$\text{Average Speed} = \bar{s} = \frac{\Delta s}{\Delta t}$$

For an object traveling in one direction along a straight line, the average speed is the magnitude of the average velocity.

Problem: A student runs for 43 minutes at an average speed of 2.22 m/s. How far did she run?

Given: $\bar{s} = 2.22$ m/s
$\Delta t = 43$ min

Want: Δs

What principle and equation relates average speed to distance?

$$\bar{s} = \frac{\Delta s}{\Delta t}$$

$\Delta s = (\bar{s})\ (\Delta t)$

$= (2.22 \text{ m/s})(43 \text{ min})(60 \text{ s/1 min})$

$= 5700$ m

Does this seem reasonable? 5700 m = 3.6 miles

Interactive Question

You jog around a 400 m track in 100 seconds, returning to the place where you started. Which of the following statements is true?

A) Your average speed and average velocity are the same, and neither is zero.
B) Your average speed and average velocity are the same, and both are zero.
C) Your average velocity is zero, and your average speed is 4 m/s.
D) Your average speed is zero, and your average velocity is 4 m/s.

Interactive Question Answer

You jog around a 400 m track in 100 seconds, returning to the place where you started. Which of the following statements is true?

C) Your average velocity is zero, and your average speed is 4 m/s.

Average Speed Reviewed

Hint for Homework Problem 14 (Chapter 2 Problem 70)

$$\text{Average Speed} = \bar{s} = \frac{\Delta s}{\Delta t}$$

To find the average speed you need to find the total distance traveled divided by the total time, not just take the average of different average speeds.

Interactive Question

Which physical quantity is not correctly paired with its SI unit and dimension?

Quantity	**Unit**	**Dimension**
A) velocity	m/s	[L]/[T]
B) path length	m	[L]
C) speed	m/s	[L]/[T]
D) displacement	m/s^2	$[L]/[T]^2$
E) speed × time	m	[L]

Interactive Question Answer

Which physical quantity is not correctly paired with its SI unit and dimension?

Quantity	**Unit**	**Dimension**
D) displacement	m/s^2	$[L]/[T]^2$

Should be m [L]

E) Speed times time is (m/s)(s) = m

Instantaneous Velocity and Speed

Instantaneous velocity is defined as:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- The absolute value of the magnitude of the instantaneous velocity is the instantaneous speed.
- For example, the speedometer in your car gives your instantaneous speed, but not instantaneous velocity.

When discussing velocity and speed, we will always mean instantaneous velocity or speed, unless explicitly stated otherwise.

Interactive Question

When is the average velocity of an object equal to the instantaneous velocity?

A) This is always true.

B) This is never true.

C) This is the case when the velocity is constant.

D) This is the case only when the velocity is increasing at a constant rate.

Interactive Question Answer

When is the average velocity of an object equal to the instantaneous velocity?

C) This is the case when the velocity is constant.

Interactive Question

When is the instantaneous speed of an object equal to the magnitude of the instantaneous velocity?

A) This is always true.

B) This is never true.

C) This is the case only when the velocity is constant.

D) This is the case only when the velocity is increasing at a constant rate.

Interactive Question Answer

When is the instantaneous speed of an object equal to the magnitude of the instantaneous velocity?

A) This is always true.

Problem Solving Steps

1. Think about the problem
   A. Read the problem twice carefully.
   B. Draw a picture of the situation with given variables.
   C. Write down what the problem is asking for.
   D. Think about the physics principles and determine the approach to use.
2. Draw a "physics diagram" and define variables.
   A. Write down what is given in the problem.
   B. Determine which equations can be used.
3. Do the calculation.
   A. It is a good idea to start with the "target" variable
   B. Do algebra before using numbers
   C. Check the units.
4. Think about the answer.
   A. Is it reasonable? (Order of magnitude)