

Physics 2414, Spring 2008

Group Exercise 6, Feb 28, 2008

Name 1: _____ OUID 1: _____
Name 2: _____ OUID 2: _____
Name 3: Solution OUID 3: _____
Name 4: _____ OUID 4: _____

Section Number: _____

Circular Motion

A mass m , moving in a circle of radius r , with a speed v , experiences a centripetal acceleration towards the center of the circle. The magnitude of the centripetal acceleration is given by

$$|\vec{a}| = \frac{v^2}{r}. \quad (1)$$

The Newton's law for circular motion in the radial direction takes the form

$$|\vec{F}_{\text{net}}| = m \frac{v^2}{r} \quad (\text{in the radial direction.}) \quad (2)$$

The frequency of rotation, and period of rotation.

$$f = \text{frequency} \quad T = \text{period} \quad (3)$$

are related

$$T = \frac{1}{f} \quad f = \frac{1}{T}. \quad (4)$$

The centripetal acceleration can be written in terms of either of the above parameters,

$$|\vec{a}| = \frac{v^2}{r} = 4\pi^2 f^2 r = \frac{4\pi^2 r}{T^2}. \quad (5)$$

Problems

Curie and her Mom are having fun by swinging rocks in horizontal circles around them. As expected her Mom can swirl the rock at much higher speeds.

1. Mom's rock:

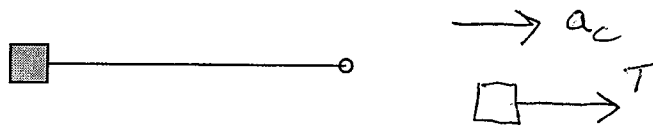
Since Curie's Mom swings the rock pretty fast, the vertical dip in the angle is very tiny. Thus we can neglect the vertical motion of her rock. We achieve this quantitatively by assuming that there is no gravitational force acting on the rock.

(a) If her Mom uses a rope of length $r = 1$ meter to swirl the rock with a tangential speed of $v = 25$ m/s, determine the frequency and period of the rock's motion.

$$f = \frac{1}{T} = 4 \text{ Hz} \quad (6)$$

$$T = \frac{2\pi r}{v} = \frac{2\pi \cdot (1\text{m})}{25 \text{ m/s}} = .25 \text{ s} \quad (7)$$

(b) Draw the free body diagram for her Mom's rock. Remember we are neglecting the gravitational force.



(c) Write Newton's law for her Mom's rock, and thus get the expression for the tension in the string.

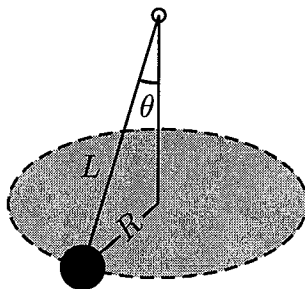
$$T = \frac{mv^2}{r} \quad (8)$$

(d) Calculate the tension in the string if $v = 25$ m/s, radius $r = 1$ meter, and mass $m = 100$ grams.

$$|\vec{T}| = \frac{(0.1 \text{ kg})(25 \text{ m/s})^2}{1 \text{ m}} = \boxed{62.5 \text{ N}} \quad (9)$$

2. Curie's rock - Conical Pendulum:

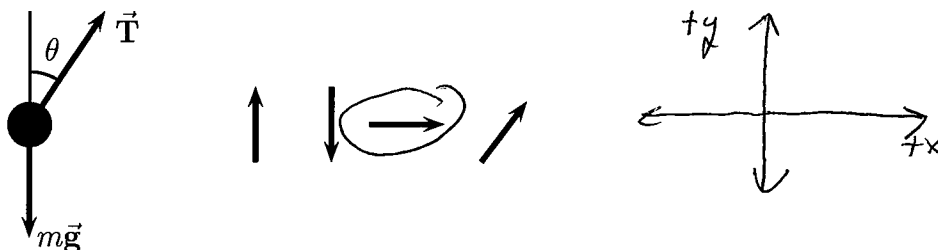
Since Curie is just learning to swirl a rock she can not do it as fast as her Mom. As a consequence the motion of the rock makes a right circular cone around her. We can not neglect gravity now.



(a) The rope attached to Curie's rock sweeps over a surface of a cone. The rock's motion is along a horizontal circle. What is the radius R of this circle in terms of the length L of the rope and angle θ the rope makes with the vertical?

$$R = L \sin \theta \quad (10)$$

(b) The free body diagram for Curie's rock is drawn below. What is the direction of the centripetal acceleration of Curie's rock. Since it is misleading to draw the acceleration vector on the free body diagram, do not draw it on the free body diagram. Circle the correct direction of the centripetal acceleration from the choices given below. Also draw an appropriate coordinate system for this problem. (Hint: Choose one of the axis along the direction of centripetal acceleration.)



(c) Write Newton's laws for Curie's rock in the radial and vertical direction. (Caution: Be careful while deciding on the radius of the

circle - refer eqn. (10). $\vec{F}_{net} = m \vec{a}$

Newton's law: $\vec{T} + m\vec{g} = m\vec{a}_c$ (11)

Horizontal component: $T \sin \theta = \frac{mv^2}{R}$ (12)

Vertical component: $T \cos \theta = mg$ (13)

(d) Determine the expression for the tension in the string.

$$|\vec{T}| = \frac{mg}{\cos \theta} \quad (14)$$

(e) Determine the expression for the linear speed of Curie's rock.

put 14 into 12 $\left(\frac{mg}{\cos \theta}\right) (\sin \theta) = \frac{mv^2}{R}$

$$v = \sqrt{gR \tan \theta} = \sqrt{gL \sin \theta \tan \theta} \quad (15)$$

(f) If the rope attached to Curie's rock is 0.5 meter long, the mass of the rock is 50 grams, and if the angle the rope makes with the vertical is 30° , find the tangential speed of Curie's rock and the tension in her rope.

$$|\vec{T}| = \frac{(0.05 \text{ kg}) \cdot 9.8 \text{ m/s}^2}{\cos 30^\circ} = 0.57 \text{ N} \quad (16)$$

$$v = \sqrt{(9.8 \text{ m/s}^2) \cdot (0.5 \text{ m}) (\sin 30^\circ) (\tan 30^\circ)} = \boxed{1.02 \text{ m/s}} \quad (17)$$

(g) Find the frequency f and period T of Curie's rock.

$$f = \frac{1}{T} = \boxed{0.76 \text{ Hz}} \quad (18)$$

$$T = \frac{2\pi L \sin \theta}{v} = \frac{2\pi (0.5 \text{ m}) \sin 30^\circ}{1.02 \text{ m/s}} \quad (19)$$

$$= \boxed{1.31 \text{ s}}$$