Physics 2414, Spring 2008
Group Exercise 11, Apr 24, 2008

Name 1: ________ OUID 1: ________

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Name 4: ________ OUID 4: ________

Solutions Solutions

Section Number: ____

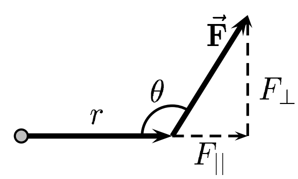
Equilibrium of rigid bodies

Torque: Torque about an axis due to a force $\vec{\mathbf{F}}$ is determined by the expression

$$\tau = rF_{\perp} = r|\vec{\mathbf{F}}||\sin\theta| \tag{1}$$

where r is the distance between the axis and the point of action of the force, and $F_{\perp}$ is the component of the force that contributes to the rotation. In the diagram shown

$$F_{\perp} = |\vec{\mathbf{F}}||\sin\theta|. \tag{2}$$



Newton's second law for translational motion of a mass m is

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} \tag{3}$$

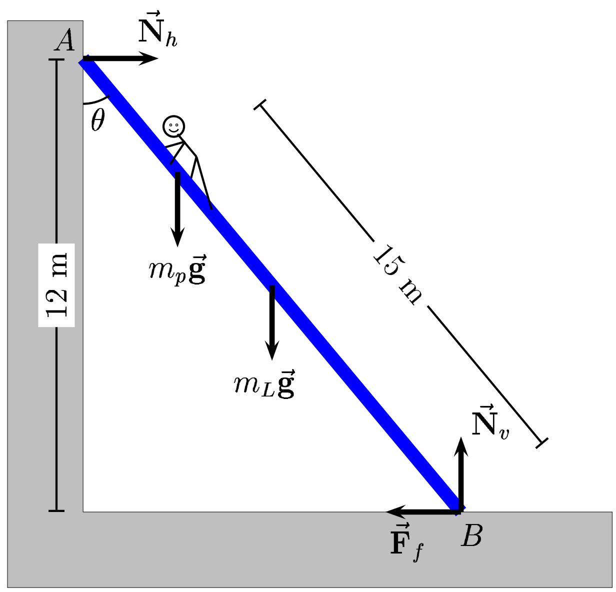
where $\vec{\mathbf{F}}_{\text{net}}$ is the sum of all the forces acting on the mass, and $\vec{\mathbf{a}}$ is the translational acceleration of the mass.
Newton's second law for rotational motion of a rigid body with rotational inertia I about an axis is

$$\tau_{\text{net}} = I\alpha \tag{4}$$

where τ_{net} is the sum of all the torques acting on the rigid body, and α is the angular acceleration of the rigid body about the axis.

Problems

A ladder of length $l = 20$ meters weighing $m_L g = 500$ Newtons rests against a wall at a point $h = 12$ meters above the ground. The center of mass of the ladder is at the center of the ladder. A man weighing $m_p g = 800$ Newtons climbs a distance $x = 15$ meters up the ladder. The friction on the floor keeps the ladder from slipping. Assume that the wall is frictionless.



1. *Newton's law for translational equilibrium:*

The Newton's law for the translational equilibrium in our problem is described by the vector equation

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} \quad (\text{equilibrium} \Rightarrow \vec{\mathbf{a}} = 0) \tag{5}$$

$$\vec{\mathbf{N}}_h + m_p\vec{\mathbf{g}} + m_L\vec{\mathbf{g}} + \vec{\mathbf{F}}_f + \vec{\mathbf{N}}_v = 0 \tag{6}$$

(a) Write down the y-component of eqn. (6) and thus determine the magnitude of the vertical normal force $\vec{\mathbf{N}}_v$.

$$0 - m_p g - m_L g + 0 + N_v = 0$$
$$N_v = m_p g + m_L g = 800 + 500 = 1300 \text{ Newtons} \tag{7}$$

(b) Write down the x-component of eqn. (6) and thus get an expression relating N_h and F_f.

$$+N_h + 0 + 0 - F_f + 0 = 0$$
$$N_h = F_f \tag{8}$$

2. *Newton's law for rotational equilibrium:*

The Newton's law for the rotational equilibrium in our problem is described by the vector equation

$$\vec{\tau}_{\text{net}} = I\vec{\alpha} \quad (\text{equilibrium} \Rightarrow \vec{\alpha} = 0) \tag{9}$$

$$\vec{\tau}_h + \vec{\tau}_p + \vec{\tau}_L + \vec{\tau}_f + \vec{\tau}_v = 0 \tag{10}$$

(a) In principle you can choose any point to be the axis about which you will calculate the torque. In our problem choosing point 'B' as the axis simplifies the algebra involved in the calculation, because more number of torques listed in eqn. (10) go to zero under this choice.

(a.1) Which of the torques listed in eqn. (10) go to zero if you choose point 'B' as your axis?

√ (i) τ_f and τ_v

(ii) τ_L and τ_p

(a.2) Which of the torques listed in eqn. (10) go to zero if you choose point 'A' as your axis?

√ *(i)* τ_h *(iii)* τ_L *(v)* τ_v

(ii) τ_p *(iv)* τ_f

(b) Determine the magnitude of the torques $\vec{\tau}_f$ and $\vec{\tau}_v$ about the point 'B'.

$$\vec{\tau}_f = 0 \tag{11}$$
$$\vec{\tau}_v = 0 \tag{12}$$

(c) Determine the magnitude and direction of the torque due to the force $m_L\vec{\mathbf{g}}$.

$$\begin{aligned}\vec{\tau}_L &= r_L\ m_L g\ |\sin\theta_L| &(13)\\ &= 10\times 500\times|\sin\theta| \qquad (\theta=\cos^{-1}\frac{12}{20}=53.13^o) &(14)\\ &= 4000 \text{ Newton-meter}\end{aligned}$$

direction: (i) clockwise (-ve) ✓(ii) anticlockwise (+ve)

(d) Determine the magnitude and direction of the torque due to the force $m_p\vec{\mathbf{g}}$.

$$\begin{aligned}\vec{\tau}_p &= r_p\ m_p g\ |\sin\theta_p| &(15)\\ &= 15\times 800\times|\sin\theta| \qquad (\theta=53.13^o) &(16)\\ &= 9600 \text{ Newton-meter}\end{aligned}$$

direction: (i) clockwise (-ve) ✓(ii) anticlockwise (+ve)

(e) Determine the magnitude and direction of the torque due to the force $\vec{\mathbf{N}}_h$.

$$\begin{aligned}\vec{\tau}_h &= r_h\ N_h\ |\sin\theta_h| &(17)\\ &= 20\times N_h\times|\sin(90-\theta)| \qquad (\theta=53.13^o) &(18)\\ &= 12N_h \text{ Newton-meter}\end{aligned}$$

direction: ✓ (i) clockwise (-ve) (ii) anticlockwise (+ve)

(f) Use the results in (b) to (e) above in eqn. (10) to determine the magnitude of the horizontal normal force.

$$0+9600+4000+0-12N_h=0$$

$$N_h=\frac{9600+4000}{12}=1133.3 \text{ Newtons} \qquad (19)$$

3. *Will the ladder slip?*

(a) What is the frictional force acting on the ladder? (Hint: use the results in eqn. (8) and eqn. (19).)

$$\begin{aligned}F_f &= N_h &\text{(using eqn. (8))}\\ F_f &= 1133.3 \text{ Newtons} &\text{(using eqn. (19))} \qquad (20)\end{aligned}$$

(b) Recollect that the force of static friction is defined as

$$F_f \leq \mu_s N_v. \tag{21}$$

In our problem since ($F_f = N_h$, using the result in eqn.(8)) we can write,

$$N_h = F_f \leq \mu_s N_v. \tag{22}$$

Using this determine the minimum coefficient of static friction the floor should have to keep the ladder from slipping when the man is $x = 15$ meters up the ladder.

$$\begin{aligned} N_h &= 1133.3 \text{ Newtons} && \text{(using eqn. (19))} \\ N_v &= 1300 \text{ Newtons} && \text{(using eqn. (7))} \end{aligned}$$

Thus using the above in eqn. (22) we have

$$\begin{aligned} 1133.3 &\leq \mu_s \times 1300 \\ 0.87 = \frac{1133.3}{1300} &\leq \mu_s \\ \Rightarrow \quad \mu_{s_{\min}} &= 0.87 \end{aligned} \tag{23}$$