Physics 2414, Spring 2008
Group Exercise 9, Apr 3, 2008

Name 1: _____ OUID 1: _____

Name 2: Solutions OUID 2: _____

Name 3: _____ OUID 3: Solutions

Name 4: _____ OUID 4: _____

Section Number: ____

Conservation of Linear Momentum

Conservation of Energy: The sum of change in kinetic energy ΔK and the change in potential energy ΔU equals the work done by the non-conservative forces.

$$\Delta K + \Delta U = \Sigma\, W_{nc}. \tag{1}$$

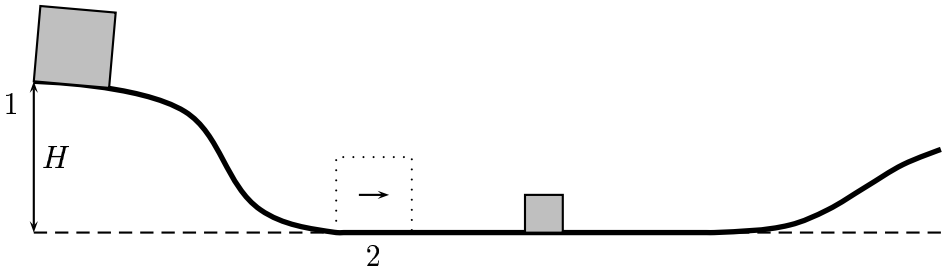
Momentum: A mass m moving with a velocity $\vec{\mathbf{v}}$ has a momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ associated with it.

Conservation of linear momentum: If there is no external forces acting on a system, the total linear momentum of the system is conserved.

$$\text{If} \quad \vec{\mathbf{F}}_{\text{net}} = 0, \quad \vec{\mathbf{p}}_{f\text{tot}} = \vec{\mathbf{p}}_{i\text{tot}}. \tag{2}$$

Problems

A skier with mass $M_1 = 75$ kg starts from rest at point '1' at a height $H = 50$ meters from the ground. Assume frictionless surfaces all throughout this problem.



1. *Before collision:*

Find the velocity v_2 of the skier when he reaches point '2'.

$$\Delta K^{12} + \Delta U^{12} = \Sigma\ W_{nc}^{12} \tag{3}$$

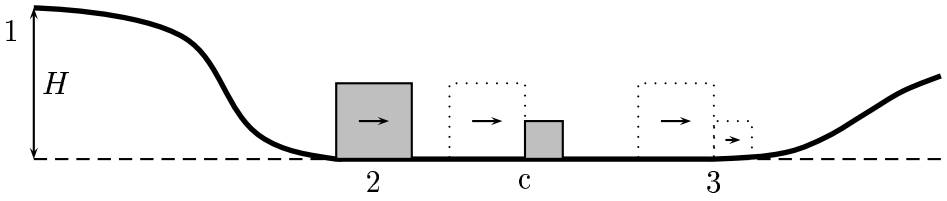
$$(\frac{1}{2}M_1 v_2^2 - 0) + (0 - M_1 g H) = 0 \tag{4}$$

$$v_2 = \sqrt{2gH} \tag{5}$$

$$= \sqrt{2 \times 9.8 \times 50} = 31.3\text{m/s} \tag{6}$$

2. *Conservation of linear momentum:*

The skier keeps moving on the horizontal frictionless surface with velocity v_2. The skier collides into a box of mass $m_2 = 25$ kg at point 'c'. After the collision the skier grabs the box and then moves with the box as a single entity with velocity v_3.



(a) What is the initial momentum $\vec{\mathbf{p}}_{1i}$ of the skier just before the collision?

$$\vec{\mathbf{p}}_{1i} = M_1 v_2 = 75 \times 31.3 = 2347 \text{ N sec} \tag{7}$$

(b) What is the initial momentum $\vec{\mathbf{p}}_{2i}$ of the box just before the

collision?

$$\vec{\mathbf{p}}_{2i} = 0 \qquad \text{since box starts from rest.} \tag{8}$$

(c) What is the final momentum $\vec{\mathbf{p}}_{1f}$ of the skier just after the collision in terms of v_3?

$$\vec{\mathbf{p}}_{1f} = M_1 v_3 = 75 \times v_3 \tag{9}$$

(d) What is the final momentum $\vec{\mathbf{p}}_{2f}$ of the box just after the collision in terms of v_3?

$$\vec{\mathbf{p}}_{2f} = m_2 v_3 = 25 \times v_3 \tag{10}$$

(e) Is there any net external force acting on the system consisting of (skier+box)? (Is the system accelerating? Remember Newton's law, $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$.)

$$\vec{\mathbf{F}}_{\text{net}} = 0 \tag{11}$$

(f) During the collision the masses exert forces on each other. The skier exerts a force $\vec{\mathbf{F}}_{21}$ on the box and the box exerts a force $\vec{\mathbf{F}}_{12}$ on the skier. What can you conclude about the magnitude of the forces $|\vec{\mathbf{F}}_{21}|$ and $|\vec{\mathbf{F}}_{12}|$?

(i) $|\vec{\mathbf{F}}_{21}| > |\vec{\mathbf{F}}_{12}|$

√ (ii) $|\vec{\mathbf{F}}_{21}| = |\vec{\mathbf{F}}_{12}|$

(iii) $|\vec{\mathbf{F}}_{21}| < |\vec{\mathbf{F}}_{12}|$

Thus conclude that the internal forces cancel each other.

(g) Using (a) to (f) determine the total initial momentum $\vec{\mathbf{p}}_{i\text{tot}}$ and the total final momentum $\vec{\mathbf{p}}_{f\text{tot}}$.

$$\vec{\mathbf{p}}_{i\text{tot}} = \vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} \qquad = 2347 + 0 = 2347 \text{ N sec} \tag{12}$$

$$\vec{\mathbf{p}}_{f\text{tot}} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \qquad = 75 \times v_3 + 25 \times v_3 = 100 \times v_3 \tag{13}$$

(h) Conservation of linear momentum states that

$$\text{if} \quad \vec{\mathbf{F}}_{\text{net}} = 0, \quad \vec{\mathbf{p}}_{f\text{tot}} = \vec{\mathbf{p}}_{i\text{tot}}. \tag{14}$$

Use this to determine the velocity v_3 of (skier+box) just after the collision.

$$\vec{\mathbf{p}}_{f\text{tot}} = \vec{\mathbf{p}}_{i\text{tot}} \quad (15)$$

$$100 \times v_3 = 2347 \quad (16)$$

$$v_3 = 23.5 \text{ m/s} \quad (17)$$

3. *After collision:*

The skier and the box keep moving up the hill as a single entity. They rise to a maximum height h above the horizontal surface.

(a) Find the height h.

$$\Delta K^{34} + \Delta U^{34} = \Sigma W_{nc}^{34} \quad (18)$$

$$(0 - \frac{1}{2}(M_1 + m_2)v_3^2) + ((M_1 + m_2)gh - 0) = 0 \quad (19)$$

$$h = \frac{v_3^2}{2g} = \frac{23.5^2}{2 \times 9.8} = 28.2 \text{ meters} \quad (20)$$

4. *Analysis:*

(a) After the collision the skier and the box are moving as a single entity with velocity v_3 at point '3'. Pick the correct answer.

√ (i) Momentum of the skier is greater than the momentum of the box at point '3'.

(ii) Momentum of the skier is less than the momentum of the box at point '3'.

(iii) Momentum of the skier is equal to the momentum of the box at point '3'.

(b) Is the kinetic energy conserved between point '2' and point '3'?

(i) Yes

√ (ii) No

Explain your answer.

Explaination: Between point '2' and point '3' we have

$$\Delta K^{23} + \Delta U^{23} = \Sigma W_{nc}^{23}. \tag{21}$$

$\Delta U^{23} = 0$ since the points are at the same height. Work done by the contact forces during the collision contributes to W_{nc}^{23}. Thus, since $W_{nc}^{23} \neq 0$ the change in kinetic energy ΔK^{23} is finite.